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S.M.RYTOV

THE THERMAL NOISE THEORY, part 1.

(A discussion is given of the general electrodynamic theory of thermic electrical fluctuations, see Bibl. 1, and some results are given for its application to problems that are of interest in radio physics and radio technology.)

INTRODUCTION

As the sensitivity of radio receiving devices increased, greater and greater importance was taken on by the apparatus' own noises --- accidental electrical processes of various origins. One type of noise is electrical fluctuation (of current, voltage or charge) as caused by the thermal motion of micro-charges in the bodies, so-called thermal noises, which depend fundamentally on the temperature of the bodies. It is precisely these noises that determine the lowest level of over-all noise possible at a given temperature in the output of the receiving device. They are therefore of interest not only where they themselves are the object of observation, but also as the principal limit which radio technology approaches as its advances in the direction of lowing the factor of noise.

At the same time as the experimental discovery of thermal noises, a theory was elaborated about them, and it is this theory that we will discuss below. It should be noted immediately, however, that this theory was adapted to thermic electrical fluctuations in quasi-stationary circuits, characterized by concentrated impedances. It thus relates to devices in which the dimensions of l are extremely small in comparison with the wave length λ in the surrounding space.

Learning to use shorter and shorter wave lengths, radio technology finally arrived at the u.h.f. range, in which we are also faced with thermal noises, but where the premises of the quasi-stationary theory are no longer fulfilled since the dimensions of the new devices are comparable to λ. Together with this, improvements in receiving apparatus made it possible to pick up, in the range from

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meter-long to centimeter-long waves, thermal electromagnetic radiations issuing from external sources. As we know, on this technical basis a new science is developing, that of radioastronomy. The peculiar feature of radioastronomical apparatus is the fact that the very signal received by the device is a thermal noise, while the problem consists of measuring it on the background (actually, below the level) of the apparatus' own noises. But regardless of the "internal" or "external" origin of thermal electrical fluctuations, we must deal with the electro-magnetic field (in particular, the radiation field) in a range in which the applic-ation of the quasi-stationary theory cannot be justified without special analysis.

There exists, however, another physical theory the subject of which is also the chaotic electromagnetic field caused by the thermal fluctuations of charge and current in the bodies. This is the classical theory of thermal radiation devel-oped in the second half of the 19th century and completed with Planck's hypothesis of light quanta. Can this theory be directly applied to questions interesting radio technology? The answer here is also negative since the classical theory of thermal radiation regards the electromagnetic field in terms of approximated geometrical optics. Its laws pre-suppose that the dimensions of the bodies, the radii of their surface curvatures, etc., are very large in comparison with the wave length, which is entirely justified in the case of many optical problems but which is not now applicable to u.h.f.

Thus, until recently we had: (1) a theory of thermal electrical fluctuations for the quasi-stationary region $(l \ll \lambda)$; (2) a theory of thermal electromagnetic radiation for the region of geometrical optics $(l \gg \lambda)$; whereas in the u.h.f. radio range, $l \sim \lambda$, i.e. neither of the aforesaid conditions is fulfilled. It is understood that there are no grounds for assuming that the two extreme systems are valid for explaining the intermediate area. In other words, we are in need of a theory of thermal electrical fluctuations that is not bound up either with

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the condition of quasi-stationariness or with the approximation of geometrical optics. Naturally, this must be a statistical theory, but it must rely on general electrodynamics.

This kind of general theory of thermal electrical fluctuations, created in the recent past, encompasses questions that go far beyond the limits of radio technology. This article has as its purpose to set forth, in brief, those results of the said theory that can be of interest from the point of view of radio physics and radio technology. In parts 2 and 3 we will briefly summarize the fundamentals of the theory, whereas a number of purely physical and at times quite complex problems, which had to be dealt with in our initial work on this theme (Bibl.1), are omitted here.

1. ELECTRICAL FLUCTUATIONS IN QUASI-STATIONARY CIRCUITS

The theoretical approach to thermal electrical fluctuations in a quasi-stationary circuit has a great deal in common with the theory of Brownian movement, i.e. the chaotic movement of a particle suspended in a liquid. In both cases there take place chance changes of state in the macroscopic system (position of particle voltage in the circuit) as caused by the chaotic influence of a multitude of micro-particles in thermal motion (electrons in a conductor, molecules in a liquid). The effect on the particular macroscopic system can be well described in the form of some chance external force applied to the system and possessing definite statistical properties. This approach was borrowed by the theory of electrical fluctuations from the theory of Brownian movement (2), of course, with the replacement of the chance mechanical force by a chance electromotive force e(t). Thus, with regard to thermal fluctuations, any linear two-pole network must be represented as containing the source of the chance emf, e(t).

This approach was further developed by the work of Nyequist (3), who gave a spectral interpretation of the chance emf:

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$$e(t) = \int_{-\infty}^{+\infty} e \, e^{i\omega t} \, d\omega, \text{ if } e = e(\omega)$$

and who gave a formula for its spectral intensity. Specifically, in the interval of positive (Footn.a) frequencies from ω to $\omega + d\omega$, the spectral intensity of this emf is:

$$e_\omega^2 d\omega = \frac{2}{\pi} \, kTRd\omega \tag{1}$$

where k equals $1.38 \cdot 10^{-10}$ ergs/deg is the Boltzmann constant, T the absolute temperature of the two-pole network and R the latter's active resistance.

Formula 1 allows us to find the spectral intensities of the fluctuations of any electrical magnitude (charges, currents, voltages) in any linear, quasi-stationary circuit. To do this we need only write out the Kirchhoff equations while introducing into their right-handed sides all the chance emf's acting in the branches with active resistances. Having solved the Kirchhoff equations, it is not difficult to then calculate the spectral intensities of all current and voltages, i.e. the Kirchhoff laws for alternating currents, written out with the chance emf's give the complete theory of thermal electrical fluctuations in linear, quasi-stationary circuits.

The question of the derivation of formula 1 will not be handled here. We should only like to emphasize the possibility of describing thermal fluctuations as the result of the action of chance emf's localized in the circuit's active resistances and statistically independent for different resistances.

2. THE CHANCE SIDE FIELD. THE GENERAL THEORY OF ELECTRICAL FLUCTUATIONS

As we know, the emf in any closed circuit is the linear integral (circulation across the circuit) from the intensity of the so-called side electrical field, distributed in the conducting wires forming the circuit. The side field is introduced in electrodynamics as some equivalent magnitude allowing us to express "in electrically language" the various forces acting on the charges but not having an electromagnetic origin, or more precisely, performing their work owing to some outside source of energy -- mechanical, thermal, chemical, etc.

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It is natural to extend this general interpretation of the emf to the chance
emf causing electrical fluctuations. Here we approach the notion of the chaotic
side field localized in the matter of any body. In this field are included all ir-
regular forces that act on the micro-charges and that are tied up with the orderless
thermal motion both of these micro-charges and of all other micro-particles. The
intensity of the outside field $K(x, y, x, t)$ is, consequently, the chance function
of the point and time but depends, obviously, on the temperature and electrical
properties of the material. At the same time, by its very nature, intensity K should
simply be added to the intensity of the electrical field in the particular medium,
i.e. should be included in Ohm's law in the usual way, in its differential form:

$$i_{cond} = \sigma\,(E + K) \tag{2}$$

where i_{cond} is the current density of conductance and σ the specific electrical
conductance of the medium.

Transition from the integral emf of a to side field K distributed over the
entire volume of the body, immediately allows us to go beyond the limits of
quasi-stationary problems. By means of Ohm's differential law (formula 2), the
chance side field can be introduced into the system of general electrodynamic equa-
tions. We thus obtain Maxwell's system of nonuniform equations, the right-hand
sides of which contain the chance side field K. When solving the equations, this
field should be regarded as pre-assigned. For any system of bodies, the Maxwell
equations (together with the boundary conditions with the necessary conditions for
infinity) exactly determine the intensities E and H, i.e. they allow us to express
the components of E and H in the form of some volumetric integrals from the
components of K, which terminates the electrodynamic part of the solution of any
problem dealing with thermal electrical fluctuations. The next and last step is the
statistical one.

We are usually interested in the average energy magnitudes characterizing the
energy of the fluctuating electromagnetic field and the transfer of energy in this

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field. For example, the spectral intensity of the density of an energy stream (Umov-Pointing vector) will be:

$$S_\omega = \frac{c}{4\pi} \{ \overline{[E, H^*]} + \overline{[E^*, H]} \}$$

As was said, as a result of solving the electrodynamic problem, E and H are expressed through the K components. Consequently, to calculate the average values from the products of the E and H components (and just such products enter into the energy magnitudes) we must know the definite statistical characteristics of the side field K, and more precisely, we must know the average values of the products of the K components taken at different points of the space. Such magnitudes are called correlation functions of the K components. In the given case they are the space correlation functions. They characterize to what extent the values of the K_α component at point x, y, z and the values of the K_β component at another point x', y', z' are related statistically to each other.

A number of general considerations, together with the requirement that for rather high frequencies $(l \gg \lambda)$ the laws of the classical theory of thermal radiation remain in force, permits us to completely determine the form of the correlation functions of the K components. By virtue of this we obtain everything necessary for calculating all energy magnitudes characterizing the thermal fluctuations of an electromagnetic field and, in particular, the fluctuating wave field, i.e. the thermal radiation.

We emphasize onee again that since the electrodynamic part of the problem is solved on the basis of general field equations, the correlation between the dimensions of the bodies and the wave length is in no way limited, i.e. the solution encompasses all the diffraction phenomena occurring in the given geometrical conditions. On the other hand, in border cases of $l \gg \lambda$ or $l \ll \lambda$ (where l characterizes the size of the bodies) there naturally come into force the approximation of geometrical optics or the quasi-stationary approximation.

Evidently, of greatest interest is the application of the afore-described

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$$P_\omega = \frac{\theta}{6\pi^2} k^3 a\varepsilon''$$

This formula encompasses the transition to an ideal conductor, when $\sigma \to \infty$ and $d \to 0$. If, on the other hand, $|\varepsilon|^2$ is not great, so that $\lambda \gg a|\sqrt{\varepsilon}|$:

$$P_\omega = \frac{\theta k}{4\pi^2 a} \sqrt{\frac{d}{a|\ln(ka|2)|^3}}$$

Upon transition to an ideal dielectric ($\varepsilon'' \to 0$), the radiation power approaches zero proportionally to the conductance. The relationship to the frequency ω and to the radius a is completely different in all three cases.

3. THE CASE OF FAIRLY WELL CONDUCTING BODIES

For the majority of problems on thermal noises in the u.h.f. range, the afore-described general theory can be simplified to a considerable extent. The reason for this is that in this frequency range the skin effect is expressed very strongly even in not particularly well conducting materials, so that the dimension-less parameter kd is extremely small:

$$kd = \frac{2\pi d}{\lambda} = \sqrt{\frac{f}{\sigma}} \ll 1 \tag{5}$$

For example, in the case of a frequency $f = \frac{\omega}{2\pi} = 3,000$ Mc, we find that kd equals 0.055 even at a conductivity of 10^{12} absolute units, which is only 10 times greater than the conductivity of sea water and more than 1,000 times smaller than the conductivity of carbon. For copper ($\sigma = 5.7 \cdot 10^{17}$ abs. units) at this frequency, kd equals $7 \cdot 10^{-5}$.

As we know, if condition 5 is fulfilled in the bodies, the electromagnetic field outside the bodies is only slightly disturbed in comparison with what it would be in the case of ideally conductive bodies with the same geometric para-meters. This permits us, generally, to free ourselves from the patent incorpora-tion of the field inside the bodies and to examine only the external field interest-ing us, subjecting the latter to some approached boundary conditions that are close to the conditions on the surface of an ideal conductor. Specifically, when 5 is fulfilled and when we have a surface that is not very greatly distorted (the

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method to problems in which $l \sim \lambda$ since it is precisely in these cases that we cannot approach the problem either from the classical theory of thermal radiation or from the quasi-stationary theory, which makes use of the integral fluctuating emf. By way of an example we will give the results of a problem on the thermal radiation of an infinitely long, annular cylinder in which the radius of cross-section a can be found in any possible correlation with the wave length λ in the surrounding transparent medium (for simplicity's sake. in a vacuum).

We are interested in the power P_ω of the thermal radiation from a unit length of the cylinder in a unit interval of frequencies around ω . The solution of the problem is as follows (b)

$$P_\omega = \frac{\theta}{2\pi} \, f \left(\varepsilon, \frac{\alpha}{\lambda} \right)$$

where $\theta = kT$ and f is the highly complex function of the dielectric permeability of the cylinder and of the ratio a/λ . The relationship between the radiation and a/λ is indeed that special result that we are able to obtain here, in contrast with the result we would have obtained from the classical Kirchhoff laws on radiation, suitable where $a \gg \lambda$. We will not present the general expression for P_ω, limiting ourselves to three special cases.

With d we will designate the thickness of the skin layer in the cylinder's material:

$$d = \frac{c}{\sqrt{2\pi\sigma\omega}} \tag{3}$$

Where $a \gg \lambda \gg d$, i.e. for a thick and well-conducting cylinder, the power p radiated from a surface unit ($p_\omega = P_\omega/2_{\pi a}$) equals:

$$P_\omega = \frac{2\theta}{3\pi^2} k^3 d \left(k = \frac{\omega}{c} = \frac{2\pi}{\lambda} \right) \tag{4}$$

Such a power is radiated from a unit of any well-conducting surface in the case of such short waves that the geometrical optics approximation is justified. Formula 4 is a direct consequence of Kirchhoff's classical law. The two following formulas relate to the opposite case of a thin cylinder ($\lambda \gg a$).

If it conducts well ($a \gg d$):

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curvature radii are far greater than d), the tangential components of the external electrical and magnetic fields should be bound together on the surface by the correlations (c):

$$E_x = \frac{kd}{1-i} H_y, \quad E_y = \frac{kd}{1-i} H_x \qquad (6)$$

Here x and y are the coordinates on the surface. If the conductivity augments, $d \to 0$ and, since the magnetic field remains finite, we obtain in the limit the following conditions on the surface of an ideal conductor: $E_x = E_y = 0$.

Thus, in the case of a strong skin effect it is possible to seek only the field that is outside the bodies but that satisfies boundary conditions 6. This is a far simpler problem, and it is therefore natural to attempt, when expression 5 occurs, to profit by its advantages also in matters of electrical fluctuations.

To do this we need only introduce into 6 the surface side fluctuation field Q, which has only tangential components Q_x and Q_y. We thus obtain the boundary conditions:

$$E_x + Q_x = -\frac{kd}{1-i} H_y, \quad E_y + Q_y = \frac{kd}{1-i} H_x \qquad (7)$$

The problem of finding the fluctuating electromagnetic field E and H is now set up by means of differential equations that do not contain the side field, i.e. by means of Maxwell's uniform equations, but on the surface of the bodies there occur nonuniform boundary conditions 7.

The solution results in expressions for E and H in the external space having the form of linear functions of the components of surface side field Q. To find the average energy magnitudes, though, we now need the correlation function of Q_x and Q_y. However, to determine them we no longer require any new assumptions since the question is solved unambiguously on the basis of the already known statistical properties of the volumetric fluctuation field K. With this method we solved a problem on the thermal radiation of a sphere in which the skin layer d was small both in comparison with the wave length λ in the surrounding space and in comparison with the sphere's radius a (see Bibl. 1, paragraph 14). As for

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the correlation between a and λ , it can have any value, which consists here
again in the generalisation of the classical radiation theory, requiring fulfillment
of the condition $a \gg \lambda$.

For the spectral power radiated by the entire sphere, we obtain an expression
of the following type:

$$P_\omega = \frac{\Theta}{2\pi} f\,(kd,\ ka)$$

A picture of the function of f is given in Fig.1, in which we show the rela-
tionship between the power radiated per unit surface of the sphere ($p_\omega = \frac{P_\omega}{4\pi a^2}$)
and $ka = \frac{2\pi a}{\lambda}$ where the radius of the sphere a changes. The curve is constructed
for kg $\ll$ 0.001 (for ka < 3, curves corresponding to values of kd = 0.001 and
less coincide).

Where ka $\ll$ 1 (but of course ka $\gg$ kd since a$\gg$d) p_ω takes on the value:

$$P_\omega = \frac{3\Theta}{4\pi^2} k^3 d$$

With increase in the sphere's radius, p_ω grows and then gradually declines with
weakly expressed maximums that are slightly displaced rightwards in relation to
those ka values that correspond to the sphere's own vibrations (of the electrical
type). At large ka's, where a $\gg \lambda$, the specific power p_ω asymptotically
approaches the same value (formula 4) as we had obtained for the thick, well-con-
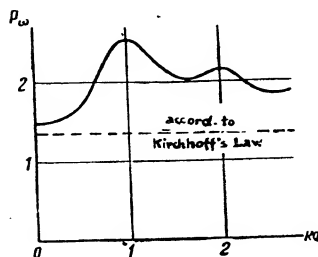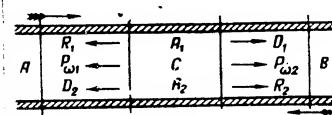ducting cylinder and which proceeds from Kirchhoff's classical law.

Fig.1

Fig.2

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From the curve in Fig.1 we see that in a small sphere (ka∼1) the radiation
per unit surface is 1½ to 2 times more intensive than in a large one (ka≫ 1).
This increase of the specific radiation power, and also of its oscillating behavior
at small ka's, is the result of diffraction of the radiated waves around the sphere.

As was already pointed out, approached boundary conditions 7 often prove to be
applicable precisely in the u.h.f. range. For the majority of questions in this
area we are interested in the wave propagation not in the free space but in feeder-
waveguides limited by metal walls or in coaxial cables. As to questions touching
upon thermal noises in the u.h.f. range, we will now turn to them. First we will
discuss the results (d) which can be obtained outside the general theory of
electrical fluctuations and then we will turn to questions for which this general
theory is indispensable.

4. THE WAVE-GUIDING FORM OF THE KIRCHHOFF LAW

Nyequist obtained formula 1 for the spectral intensity of the fluctuating
emf by examining the transfer of energy between two identical, active resistances
R joined by an ideal, two-conductor line to which they were adjusted (3). Thus,
the energy exchange occurs by means of the waves that are excited in the line by
each of the resistances and that are unreflected owing to the adjustment of the line
to the loads. The power in the frequency interval from ω to $\omega + d\omega$, sent into the
line by each resistance, can be calculated in the following way.

If at some moment we were to shorten both ends of the line, we would "catch"
in it the waves running toward each other with different frequencies ω. The
system of standing waves with frequencies from ω to $\omega + d\omega$ can be regarded
as a superposition of the line's own vibrations whose frequencies are included in

the said interval. The line's own frequencies, if its length equals $\underline{l}$, are:

$$\omega \qquad \qquad \omega_n = \frac{\pi c}{l} n \ (n = 1, 2, 3, \ldots)$$

i.e. are separated from each other by $\delta\omega = \frac{\pi c}{l}$. Assuming that $\underline{l}$ is large

enough so that $\delta\omega \ll d\omega$, we obtain the following number of inherent vibrations in the $d\omega$ interval:

$$ds = \frac{d\omega}{\delta\omega} = \frac{l\,d\omega}{\pi c}$$

In the non-quantum region, there comes into force the theorem of the regular distribution of energy by degree of freedom, according to which for each inherent vibration there is, at thermal equilibrium, an energy of $\Theta = kT$. Consequently, the energy in the line for the frequency interval of $d\omega$ is equal to:

$$\Theta\,ds = \frac{\Theta l\,d\omega}{\pi c}$$

But with acting, coordinated resistances, there is no reflection on the ends, which signifies that the resultant energy is just equal to the energy that both resistances send into the line during the run of a wave at distance $\underline{l}$, i.e. during the time $\tau = \frac{l}{c}$. Thus, each resistance sends the following energy in a second:

$$\frac{\Theta\,ds}{2\tau} = \frac{\Theta\,d\omega}{2\pi} \tag{8}$$

Possessing this expression, it is no longer difficult to obtain the spectral intensity (formula 1) for the emf acting in each resistance, but expression 8 itself is of greater interest to us for the moment.

This expression is obtained for a special kind of line and for so called main waves, in which both the electrical and magnetic fields are purely transverse. It is possible, however, to show that the same result remains in force for a line (a one-dimensional channel) of arbitrary form and for any type of wave possible in such a line, and not only for main waves, which, generally speaking, can also be impossible (for instance, in a wave guide). A proof that is analogical to the above but which takes into account the possibility of dispersion and consequently the difference between the phase and group velocities is given in Bibl. 1, paragraph 17.

Thus , an emitter having a temperature Θ and coordinated with the wave guide at frequency ω , sends, at this frequency, a power with a spectral density

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of $\frac{\varphi}{2\pi}$. If we now decide that in the presence of any uncoordinated body in the wave guide, the thermal equilibrium be preserved in each spectral interval, it is possible to obtain the power sent by the uncoordinated emitter.

Let us assume that in sections of a wave guide A and B (Fig.2) we have emitters that are completely coordinated with the wave guide at frequency ω on a wave of some type (E or H) and number (m, n), i.e. emitters that do not reflect this wave. Between them we will place an uncoordinated emitter C, and we will assume that the disagreement is, generally speaking, different on both sides of C. We will designate the coefficients of reflection, absorption and passage of C, if the wave falls to the left by R_1, A_1 and D_1, and if it falls to the right by R_2, A_2 and D_2. It is understood that $R_1 + A_1 + D_1 = R_2 + A_2 + D_2 = 1$. By $P_{\omega 1}$ and $P_{\omega 2}$ we will indicate the powers radiated by C on the said wave, on the left and right respectively.

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(The last two pages of this article are missing. Tr.)

FOOTNOTES

a) Intensities in the spectrum along positive frequencies will be noted by the subscript ω. They are twice as great as intensities in the spectrum along frequencies from $-\infty$ to $+\infty$, so that $e_\omega^2 = \overline{2ee^*}$, where e is the complex spectral amplitude of e(t); the asterisk designates a complexly conjugated magnitude and dash designates the statistical mean.

b) See Bibl. 1, paragraph 10.

c) These correlations were obtained simultaneously by A.N.Shukin (4) and M. A. Leontovich, the latter also pointing out the possibility of using them as the boundary conditions in solving marginal problems with regard to good conductors (5).

d) See, paragraphs 15 17

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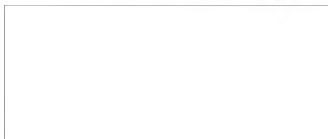
A.A.KHARKEVICH and E. L. Blokh

ON THE MAXIMUM CARRYING CAPACITY OF A COMMUNICATION SYSTEM

(From geometrical correlations a formula is developed for the maximum carrying capacity of a communication system. The final result shows that the well known Shannon formula is only valid in the limit at a signal/interference ratio approaching infinity.)

The carrying capacity of a communication system is defined as the amount of messages transmitted in the system per unit time.

The maximum carrying capacity,i.e. the maximum number of messages that can be transmitted per unit time at an unlimitedly small probability of error, is expressed by the correlation:

$$C = F \log_2 \left(1 + \frac{P}{P_{II}} \right) \qquad (1)$$

This correlation is the essence of one of the Shannon theorems (Bibl.1). In a later work (2), Shannon gives a geometrical proof for this theorem, this proof being, however, unsatisfactory.

In the present work we give a more exact proof of this theorem so as to make correlation 1 more precise.

Let us assume that a communication system carries a message that is reflected by the signal in the form of some function f(t) having a final duration T and a limited spectrum of width F. According to Kotelnikov's theorem (3), to completely define this message we need only supply the final number of separate values of function f(t), equal to:

$$n = 2FT$$

The total n of separate values of function f(t) can be represented as a point in the multi-dimensional space of n dimensions with coordinates:

$$f_k = f\left(\frac{k}{n} T\right); \quad k = 1, 2, \ldots n$$

Thus, each signal is depicted as a point or vector in the space of n dimensions. The length of this vector:

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$$f = \sqrt{\sum_{k=1}^{n} f_k^2}$$

equals the square root of the energy of the given signal.

Instead of vector f, we will examine vector:

$$\vec{y} = \frac{1}{n} \vec{f}$$

In this case the square of the length of this vector:

$$y^2 = \frac{1}{n} \sum_{k=1}^{n} f_k^2$$

will express the power of the signal. If an interference is imposed on the signal, around the point of the signal with coordinates:

$$y_k = \frac{1}{n} f_k, \quad k = 1, 2, \ldots n$$

a region of indefiniteness is formed. Expressing the interference by means of the chance vector:

$$\vec{z} = \frac{1}{\sqrt{n}} \vec{\xi}$$

with the components $\frac{1}{\sqrt{n}} \xi_k$, we obtain for the square of the length of this vector:

$$z^2 = \frac{1}{n} \sum_{k=1}^{n} \xi_k^2$$

If the interference is a white noise, then owing to the chance nature and independence of ξ_k all directions of the interference's vector are equally probable, while its length fluctuates around the value:

$$\sqrt{P_{II}} = \sqrt{\overline{\xi^2}}$$

(average power of the interference)

Thus, the region of indefiniteness around the end of the signal's vector represents a symmetrical formation in which the surfaces of equal probabilities are surfaces of rotation (we are speaking of the fall probability of the vector end of the received signal, i.e. the vector of the sum of signal and interference

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at the given point in space).

In order for the signals to be distinguished from each other it is necessary for the regions of indefiniteness of the various signals not to intersect each other. If this condition is fulfilled, the ideal receiver (in Kotelnikov's sense) will unmistakenly select the actually transmitted signal and reestablish the corresponding message.

Let us determine the maximum number of distinguishable signals for the case where n is any unlimitedly large whole number. In this case, in the expression for the square of the vector's length:

$$y^2 = \frac{1}{n} \sum_{k=1}^{n} f_k^2$$

we will regard f_k as a chance magnitude. Under these conditions, the fluctuations of the chance magnitude y^2 around the value P(P equals $\lim_{n \to \infty} \frac{1}{n} \sum f_k^2$) will decrease when n is increased, and as y^2 we can adopt the average power of the signal, P equals const. The same can be said for the vector of interference $\vec{z}$, whose square length, where n is large, can be regarded as constant and equal to the average power of interference P_{π}.

Let us now calculate the length of vector $\vec{u}$, which depicts the received signal:

$$\vec{u} = \vec{y} + \vec{z} = \frac{1}{\sqrt{n}} \vec{f} + \frac{1}{\sqrt{n}} \vec{\mathcal{E}}$$

The square length of this vector equals:

$$u^2 = \frac{1}{n} \sum_{k=1}^{n} (f_k + \mathcal{E}_k)^2 = \frac{1}{n} \sum f_k^2 + \frac{1}{n} \sum \mathcal{E}_k^2 + \frac{2}{n} \sum f_k \mathcal{E}_k$$

or

$$u^2 = P + P_{II} + \frac{2}{n} \sum f_k \mathcal{E}_k$$

But the latter item, owing to the independence of the signal and interference, is equal to zero on the average, while the fluctuations of this item decrease with increase of n. Thus, where n is rather large, with a probability unlimitedly close to unity we have:

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$$u^2 = P + P_\Pi$$

and the vector length of the received signal equals $\sqrt{P + P_\Pi}$, while the

vector length of the transmitted signal is $\sqrt{P}$ and the vector length of the inter-

ference $\sqrt{P_\Pi}$. Such a relationship between vectors reflects statistical indepen-

dence,i.e. the absence of a correlation between the signal and the interference.

Geometrically, however, this is reflected in the fact that the interference's

vector is orthogonal to the vector of the transmitted signal, as is shown in

Fig.1. Consequently, the ends of the vectors of the transmitted signals lie

on an n surface -- a uniform sphere of radius $\sqrt{P}$, while the vector ends of the

received signals lie on the surface of a sphere having a radius of $\sqrt{P + P_\Pi}$.

All directions of vector $\vec{\xi}$ in the plane normal to vector $\vec{f}$ are equally probable.

Therefore, in the case in question i.e. that of a large n, the region of indefinite-

ness degenerates into a n-dimensional circle lying on the surface of an n-dimension-

al sphere having a radius of $\sqrt{P + P_\Pi}$.
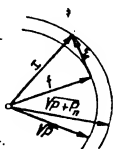
Fig.1   Fig.2.
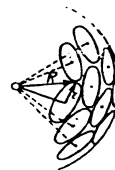
Now we can approach the problem of finding the number of distinguishable

signals in terms of a purely geometrical problem of packing the maximum number of

unintersecting circles of radius $r = \sqrt{P_\Pi}$ on the surface of a sphere of radius

$R = \sqrt{P + P_\Pi}$. The geometrical arrangement of the problem is shown in Fig.2 on

a three dimensional model.

In actuality, the problem is that of an n dimensional space, where n is a

very large number since only on this assumption do we obtain clearly defined circles

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instead of indistinct regions. This scheme can also be applied to a small n on the condition that the signal and interference powers are constant and that $P \gg P_{II}$.

For the surface of a sphere with radius R in the n-dimensional space we have the well-known formula:

$$S_n = \frac{\pi^{\frac{n}{2}} n}{\Gamma\left(\frac{n}{2} + 1\right)} R^{n-1}.$$

The area of a circle with radius r in the space equals:

$$A_n = \frac{\pi^{\frac{n-1}{2}}}{\Gamma\left(\frac{n}{2} + \frac{1}{2}\right)} r^{n-1},$$

(This is nothing other than the volume of a sphere with radius r in the space of (n − 1) dimensions.)

The number of r radius circles fitted in with the closest packing (without mutual intersection) on the surface of an R radius sphere (assuming that the number of circles is great and ignoring, for this reason, the surface's curvature) equals:

$$N = \sigma(n) \frac{S_n}{A_n} = \sigma(n) \sqrt{\pi}\, n\, \frac{\Gamma\left(\frac{n}{2} + \frac{1}{2}\right)}{\Gamma\left(\frac{n}{2} + 1\right)} \left(\frac{R}{r}\right)^{n-1},$$

where $\sigma(n)$ is the coefficient for using the area (packing coefficient).

The coefficient $\sigma(n)$ for using the area is evidently equal to the coefficient $\rho(n - 1)$ for using the volume in the space of (n − 1) dimensions when packing into it spheres having identical radii. By replacing R and r with their values, we obtain:

$$N = \sqrt{\pi}\, n\, \rho(n - 1) \frac{\Gamma\left(\frac{n}{2} + \frac{1}{2}\right)}{\Gamma\left(\frac{n}{2} + 1\right)} \left(1 + \frac{P}{P_{II}}\right)^{\frac{n-1}{2}}.$$

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At the present time there does not yet exist a complete solution of the problem dealing with the closest packing of spheres in an n-dimensional space, and consequently the exact value of the coefficient $\rho(n-1)$ is unknown. However, we have established the limits within which the packing coefficient is found; these limits are expressed by the inequalities:

$$\frac{1}{2^{n-1}} < \rho_{||} < \frac{n+2}{2^{\frac{n+2}{2}}}$$

The estimate below belongs to Minkovsky; the estimate above is given by Blichfeldt (4). Substituting the aforesaid limits into the formula for N logarithming and throwing out the terms that are small in comparison with n (where $n \to \infty$), we obtain for the limit carrying capacity:

$$F \log_2 \frac{1}{4} \left(1 + \frac{P}{P_{||}}\right) < C_m < F \log_2 \frac{1}{2} \left(1 + \frac{P}{P_{||}}\right)$$

or

$$F \left[\log_2 \left(1 + \frac{P}{P_{||}}\right) - 2 \right] < C_m < F \left[\log_2 \left(1 + \frac{P}{P_{||}}\right) - 1 \right] \qquad (3)$$

Thus, the limit carrying capacity is in any case smaller than that determined by the Shannon formula. Only where $\frac{P}{P_{||}} \to \infty$ do formulas 3 and 1 give coinciding results; here, of course, the unit in the rounded parentheses can be ignored since for this limit case:

$$C \to F \log_2 \frac{P}{P_{||}} \qquad (4)$$

Let us now examine the case where the number, N, of different signals exceeds the limit number. Under these conditions, the regions of indefiniteness (the circles of radius r) begin to intersect, which leads to an unequal to zero probability of error in receiving and reestablishing the transmitted message. The probability of error in this case is equal to the ratio of lengths of these circles (Fig.3a). Evidently, transmitting will become impossible when the r radius circles completely (without free gaps) cover the entire surface of the R radius

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sphere, since in these conditions the probability of error will be equal to unity
(Fig.3b) The number of circles completely covering the surface of the sphere
is expressed by the formula:

$$M = \sqrt{\pi}_n \delta \ (n - 1) \frac{\Gamma\left(\frac{n}{2} + \frac{1}{2}\right)}{\Gamma\left(\frac{n}{2} + 1\right)} \left(1 + \frac{P}{P_\Pi}\right)^{\frac{n-1}{2}}$$

where δ (n) is the packing coefficient in the case under investigation.

Fig.3.

a)   b)
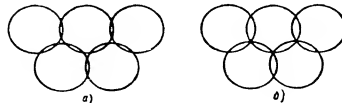
In recent articles (5,6) we were given the lower estimate of δ(n); it would
seem that where n is large this estimate is expressed as a constant magnitude,
slightly larger than unity. Therefore, where $n \to \omega$, we can write:

$$M < B \ \sqrt{n} \ \left(1 - \frac{P}{P_\Pi}\right)^{\frac{n}{2}}$$

where B is a constant multiplier. By logarithming and throwing out the slowly
augmenting terms, we obtain for the carrying capacity precisely the Shannon cor-
relation 1. However, as we see, this correlation obtains a completely different
meaning: it expresses the carrying capacity with a probability of error that is
unlimitedly close to unity.

On the other hand, with increase of P/P_Π , the limits in formula 3 and the
value of C from formula 1 unlimitedly approach the value expressed by the limit
formula 4. Following from this is the final conclusion, that under conditions of
unlimited increase of P/P_Π, the limit carrying capacity tends toward the value
expressed by formula 4; upon reaching this value, the probability of error jumps
from 0 to 1.

All the foregoing discussions and results relate to the case of $n \to \omega$.
Since n equals 2FT, at a pre assigned band F this signifies that we are dealing
with rather long message segments, while assuming that the transmitter codes this

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entire segment in the optimum manner and that the receiver reestablishes this section in picking up the complete corresponding signal.

SUPPLEMEN:T

When examining the theory expounded above, there arises an obvious question as to the correlations at which we have the right to represent the diffuse region of indefiniteness as a distinctly limited area. In other words, at what values of n do the fluctuations of the vector length of the interference become small enough.

We will state the problem in the following manner: we must find as the function of n the interval in which the vector length of the interference is included with the pre-assigned probability. Geometrically, this task leads us to find the width of the ring representing the cross section of the region of indefiniteness by means of the plane normal to the vector of the transmitted signal; the ring constricts with increase of n, and in the limit becomes the circle which figured in all the aforegoing constructions.

For the square of the vector length of the interference we had:

$$z^2 = \frac{1}{n} \sum_{k=1}^{n} \mathcal{E}_k^2$$

The distribution for z is known; it is the so called χ^2 distribution (Footn.a):

$$\varphi(y) = \frac{\sqrt{2n}}{\Gamma\left(\frac{n}{2}\right)} \left(y \frac{\sqrt{n}}{\sqrt{2}}\right)^{n-1} e^{-\frac{n}{2}y^2}$$

It is presupposed that the ξ_k are independent and distributed according to the normal law with parameters:

$$\overline{\mathcal{E}} = 0, \quad \overline{\mathcal{E}^2} = P_n$$

The integral law of distribution expressing the probability of whether the magnitude of z will be in the interval between 0 and y is given by the formula:

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$$\psi\ (y)\ =\frac{2^{1-\frac{n}{2}}}{\Gamma\left(\frac{n}{2}\right)}\ \int\limits_{0}^{\sqrt{n}y}x^{n-1}e^{-\frac{1}{2}x^2}\ dx$$

This integral is not expressed in certain functions. But it can be shown (and owing to the lack of space we do not give the proof here) that where n with precision up to terms that decrease no slower than $n^{-\frac{1}{2}}$ the above distribution can be replaced with the normal one:

$$\psi_n\ (y)\ =\frac{1}{\sqrt{2\pi}}\int\limits_{\sqrt{\frac{n}{2}}(1-y^2)}^{\sqrt{\frac{n}{2}}}e^{-\frac{1}{2}x^2}\ dx$$

We can judge as to the degree of approximation obtained through the use of this formula from the following figures:

n	2	8	18	28
$\dfrac{\psi\ (1)\ -\ \psi_n\ (1)}{\psi\ (1)}$	0.46	0.14	0.084	0.065

Let us now find the probability which the investigated chance magnitude might fall into the interval $y_1 < y < y_2$, where:

$$y_1^2\ =\ 1\ -\ \varepsilon,\ y_2^2\ =\ 1\ +\ \varepsilon\ \ (y_2\ -\ y_1\ =\ \sqrt{1\ +\ \varepsilon}\ -\ \sqrt{1\ -\ \varepsilon}\ \approx\ \varepsilon)$$

The sought for probability is expressed by the correlation:

$$p\ =\ \psi_n\ (y_2)\ -\ \psi_n\ (y_1)\ =\ \frac{1}{\sqrt{2\pi}}\ \int\limits_{-\varepsilon\sqrt{\frac{n}{2}}}^{\varepsilon\sqrt{\frac{n}{2}}}e^{-\frac{1}{2}x^2}\ dx\ =\ 2\Phi\left(\varepsilon\sqrt{\frac{n}{2}}\right)$$

where Φ is the symbol of the Laplace function (integral of probabilities). Pre-assigning a probability p we can find the relative width of the ring ε (i.e. the ratio of the width of the ring to the radius). Thus, where p equals 0.99, we obtain the following figures:

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n	10^2	10^3	10^4	10^5	10^6
ε	0.368	0.116	0.0368	0.0116	0.00368

The width of the ring decreases as $1/\sqrt{n}$

In evaluating these figures it should be remembered that for a telephone signal (F $\approx$ 5 Kc) having a duration of one second, we have:

$$n = 2FT = 10^4$$

For a television signal (F = 5 Mc) at the same duration, n is already 10^7.

Article received by the Editors on December 15, 1954.

FOOTNOTES

a) See B. V. Gnedenko. "Kurs teorii veroyatnostei" ("Course in the Theory of Probability"), pp.125 127.

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K. I. CHERNE

ON THE THEORY OF A SINGLE-TUBE R AND C GENERATOR

(A study is made of the single tube R and C generator. An equation of free
oscillations is advanced in a generalized circuit which is used to analyze a
number of special diagrams for single tube R and C generators, making it possible
to establish some general rules.)

1. INTRODUCTION

The generator examined in this article consists of a single tube amplifier
whose output is connected to the input through a quadripole consisting of an
R and C (Fig.1).

Such generators, as we know, are of practical interest when we need low
power but rather stable and small sized sources of sinusoidal voltage. Such
generators were first suggested by V. I. Siforov (Footn. a).

We find in technical literature several diagrams of single tube R and C
generators (Bibl. 1, 2, 3); the authors of these works limited themselves to a
study of diagrams in which the quadripole in the feedback circuit consists of
three or four half T shaped quadripoles of the type shown in Fig.2a or Fig.2b.

In this article we examine the general case where the quadripole in the
feedback circuit of the generator consists of any number of identical half T
shaped quadripoles of the type shown in Fig.2a or Fig.2b joined in series (cascade)
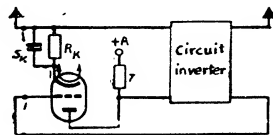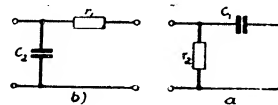
Fig.1 (Feedback circuit)   Fig.2

To study this general scheme at work we use the matrix theory of quadripoles
(4) and the technique of analyzing the stability of linear systems as developed

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by E.V.Zelyakh (see Appendix I). This made it possible to avoid the complicated computations that are inevitable when studying complex circuits by the usual method of contour currents or by the method of nodal voltages, and also to obtain with comparative ease a compact formula expressing the self excitation conditions of such tube generators.

2. DERIVATION OF FREE OSCILLATION EQUATION

Let us assume that in the diagram of Fig.1, the feedback circuit represents n cascade connected, identical, half T-shaped quadripoles, shown in Fig.3. Let us develop the free oscillations equation of this diagram. For this we must first find the parameter A_{11} of the quadripole obtained after dividing the diagram of Fig.1 at points 1 - 1 (Fig.4).
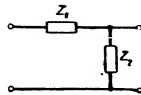
Fig.3

This quadripole represents a cascade connection of two quadripoles, the first being the amplifying cascade and the second consisting of n cascade-connected, identical, half T-shaped quadripoles as in Fig.3.

If we ignore the resistor in the cathode's circuit, parameters coefficients matrix of the first quadripole will equal (4):

$$\| \alpha \| = \left\| \begin{array}{cc} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{array} \right\| = \left\| \begin{array}{cc} -\dfrac{1}{K} & -\dfrac{1}{S} \\ 0 & 0 \end{array} \right\|, \tag{1}$$

where S is the grid plate transconductance of the tube, and K the amplification of the cascade:

$$K = \mu \, \frac{r}{r + R_i} . \tag{2}$$

Here R_i is the internal resistance of the tube, and μ the latter's amplification factor.

The parameters coefficients matrix of the second quadripole equals:

$$\| \alpha' \| = \left\| \begin{array}{cc} \alpha'_{11} & \alpha'_{12} \\ \alpha'_{21} & \alpha'_{22} \end{array} \right\| = \| a \|^n, \tag{3}$$

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where $\|a\|$ is the parameters-coefficients matrix of the quadripole in fig.3:

$$\|a\| = \left\| \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{matrix} \right\| = \left\| \begin{matrix} 1 + \dfrac{Z_1}{Z_2} & Z_1 \\ \dfrac{1}{Z_2} & 1 \end{matrix} \right\|. \tag{4}$$
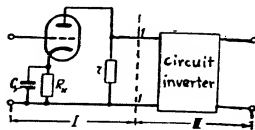
The matrix $\|A\|$ of the quadripole in fig.4 equals the product of matrixes $\|a\|$ and $\|a'\|$:

$$\|A\| = \left\| \begin{matrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{matrix} \right\| = \left\| \begin{matrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{matrix} \right\| \cdot \left\| \begin{matrix} \alpha'_{11} & \alpha'_{12} \\ \alpha'_{21} & \alpha'_{22} \end{matrix} \right\|, \tag{5}$$

or, incorporating formulas 3 and 4:

$$\|A\| = \left\| \begin{matrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{matrix} \right\| \cdot \left\| \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{matrix} \right\|^n. \tag{6}$$

<u>Fig.4</u>   (Feedback circuit).

To obtain the free oscillations equations we must determine the parameter of A_{11}. As we see from formula 5, it equals:

$$A_{11} = \alpha_{11}\alpha'_{11} + \alpha_{12}\alpha'_{21}. \tag{7}$$

The parameters of a_{11} and a_{12} are known from formula 1, while the parameters of a'_{11} and a'_{21} as we see from formula 3, are elements of the matrix obtained after raising to the nth power matrixes $\|a\|$ from the diagram of fig.3.

It is easiest to raise a matrix to a power if we first bring this matrix

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to its diagonal form. After doing this to matrix $\|a\|$, and after raising it to the nth degree we obtain (see Appendix II):

$$\| a \|^n = \left\| \begin{array}{cc} \alpha_{11}^{'} & \alpha_{12}^{'} \\ \alpha_{21}^{'} & \alpha_{22}^{'} \end{array} \right\| =$$

$$= \left\| \begin{array}{cc} \lambda_1^n - \dfrac{\lambda_2^n(\lambda_2-1)}{\lambda_1-1} & -\dfrac{(\lambda_2-1)\left(\lambda_1^n+\lambda_2^n\right)}{a_{21}} \\ \dfrac{\left(\lambda_1^n - \lambda_2^n\right)a_{21}}{\lambda_1-1} & \lambda_2 - \dfrac{\lambda_1^n(\lambda_2-1)}{\lambda_1-1} \end{array} \right\| \cdot \dfrac{\lambda_1-1}{\lambda_2-1} . \qquad (8)$$

Here λ_1 and λ_2 are the proper values of matrix $\|a\|$:

$$\lambda_1 = 1 + \frac{Z_1}{2Z_2} + \sqrt{\left(1 + \frac{Z_1}{2Z_2}\right)^2 - 1} , \qquad (9)$$

$$\lambda_2 = 1 + \frac{Z_1}{2Z_2} - \sqrt{\left(1 + \frac{Z_1}{2Z_2}\right)^2 - 1} . \qquad (10)$$

Formulas 9 and 10 are obtained from formulas a and b of Appendix II after substituting into them the values of the matrix $\|a\|$ elements from formula 4.

By substituting into formula 7 the values of a_{11} and a_{12} from formula 1 and a'_{11} and a'_{21} from formula 8, we find the sought for parameter-coefficient of A_{11}:

$$A_{11} = \frac{\left(\lambda_1^n - \lambda_2^n\right)(1-q) - \left(\lambda_1^{n+1} - \lambda_1^{n+1}\right)}{K\,(\lambda_1 - \lambda_2)} , \qquad (11)$$

where

$$q = a_{11}\frac{K}{S} = \frac{rR_i}{Z_2(r+R_i)} . \qquad (12)$$

Making the expression for A_{11} equal to unity, we obtain the free oscillations equation in the following form:

$$K = \frac{\left(\lambda_1^n - \lambda_2^n\right)(1-q) - \left(\lambda_1^{n+1} - \lambda_2^{n+1}\right)}{\lambda_1 - \lambda_2} . \qquad (13)$$

2. ANALYSIS OF PARTICULAR CASES

Let us first examine the case in which, in the diagram of fig.3:

and $\quad Z_1 = \dfrac{1}{i\,\omega\,C_1}$:

$$Z_2 = r_2,$$

which corresponds to the diagram of fig. 2a.

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which corresponds to the diagram of fig.2a.

Then from formula 9 we get:

$$\lambda_1 = 1 - i\,\frac{p_1}{2} + \sqrt{\left(1 - i\,\frac{p_1}{2}\right)^2 - 1}\,, \qquad (14)$$

$$\lambda_2 = 1 - i\,\frac{p_1}{2} - \sqrt{\left(1 - i\,\frac{p_1}{2}\right)^2 - 1}\,, \qquad (15)$$

where

$$p_1 = \frac{1}{\omega\,C_1 r_3}. \qquad (16)$$

From formula 12, in this case, we get:

$$q = \frac{r R_l}{r_3\,(r + R_l)}. \qquad (17)$$

1. Let n equal 1. In this case the diagram of fig.1 becomes the diagram of fig.5, for which, from expressins 13 to 15 we get:

$$K = -(1 + q) + i\,p_1. \qquad (18)$$

Since K is substantial, ip_1 equals 0 or ω equals ∞ and K equals $-(1 / q)$.

But K should be positive (see equality 2). Therefore, this type of circuit will not be self-exciting, i.e. the circuit of fig.5 cannot be a generator. In an analogical way it can be shown that if n equals 2, the diagram of fig.1 will also not be self-exciting.

2. Let n equal 3. In this case the diagram of fig.1 becomes the diagram of fig.6, for which, from expression 13 to 15 we get:

$$K = -1 + p_1^2\,(5 + q) - 3q + i\,p_1(6 + 4q - p_1^2). \qquad (19)$$

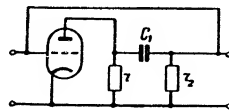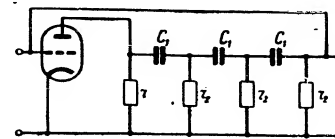Fig.5.   Fig.6.

Since K should be substantial, this equality is broken into two:

$$K = -1 + p_1^2\,(5 + q) - 3q. \qquad (20)$$

$$p_1(6 + 4q - p_1^2) = 0. \qquad (21)$$

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From formula 21 we determine p_1:

$$p_1 = \sqrt{6 + 4q} \qquad (22)$$

or, incorporating formula 16, we get:

$$f = \frac{1}{2\pi C_1 r_2 \sqrt{6 + 4q}}. \qquad (23)$$

Substituting formula 22 into formula 20, we get:

$$K = 29 + 23q + 4q^2. \qquad (24)$$

In this case $p_1 > 0$ and $K > 0$. Therefore, the diagram is self-exciting if the conditions of formulas 20 and 21 are fulfilled. Equality 24 determines the necessary minimum amplification of the cascade, while formula 23 determines the generation frequency.

If $q \ll 1$, which as we know is desirable from the point of view at the stable operation of the generator, then:

$$f \approx \frac{1}{2\pi C_1 r_2 \sqrt{6}} = \frac{0.065}{C_1 r_2} \qquad (25)$$

and

$$K = 29. \qquad (26)$$

This result coincides with the result obtained by V. I. Siforov (1) using another method.

The above examples show how, by using equalities 13-15, we can obtain the self-excitation conditions of the generator in fig.1 with different numbers of cascade-connected quadripoles from fig.2a, forming the generator's feedback circuit. The results from a number of different circuits are given in table 1. Let us now examine the case where in fig.3:

$$Z_1 = r_1 \qquad (27)$$

and

$$Z_2 = \frac{1}{i \omega C_2}, \qquad (28)$$

which corresponds to fig.2a.

In this case to the output of the tube we must connect a spacing condenser C_c and a leak resistor R_c (fig.7). If the resistance of this condenser is very

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small at the generation frequency, while the resistance of R_c is rather great, equality 11 will also be valid for the quadripole we obtain if we divide fig.7 at points 1-1. Since in this case:

$$q = a_{21} \frac{K}{S} = i \, q_2 p_2, \tag{29}$$

where

$$q_2 = \frac{rR_l}{r_1 (r + R_l)} , \tag{30}$$

while

$$p_2 = \infty \, C_2 r_1 , \tag{31}$$

equality 13 assumes the form:
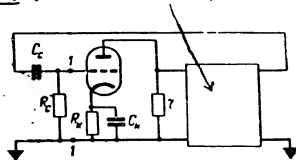
$$K = \frac{\left(\lambda_1^n - \lambda_2^n \right) (1 - i \, q_2 p_2) - \left(\lambda_1^{n+1} - \lambda_2^{n+1} \right)}{\lambda_1 - \lambda_2} , \tag{32}$$

where

$$\lambda_1 = 1 + i \frac{p_2}{2} + \sqrt{\left(1 + i \frac{p_2}{2} \right)^2 - 1} \tag{33}$$

$$\lambda_2 = 1 + i \frac{p_2}{2} - \sqrt{\left(1 + i \frac{p_2}{2} \right)^2 - 1} . \tag{34}$$

<u>Fig.7</u> (Feedback circuit, cascade connection of fig.2b)

Equalities 33 and 34 are obtained from formulas 9 and 10 after substituting into the latter two, formulas 27, 28 and 31.

Adopting various values for n, let us determine from formula 32 the generation frequency and the necessary amplification of the generators from fig.7. As in the case where the feedback circuit consisted of the cascade-connected quadripoles from fig.2a, in fig.7 generation is only possible where $n > 2$. The analysis results for a number of circuits are given in table 2.
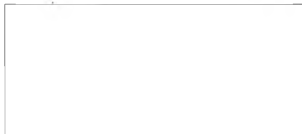
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Table 1

No.	Diagram of feedback circuit	n	q ≪ 1	
			Minimum K	Generat. freq. f in cps
1	(circuit diagram: C_1, C_1, C_1 series; z_2, z_2, z_2 shunt)	3	29	$\dfrac{1}{2\pi\sqrt{6}\,C_1 z_2} = \dfrac{0.065}{C_1 z_2}$
2	(circuit diagram: C_1, C_1, C_1, C_1 series; z_2 shunt)	4	18.38	$\dfrac{1}{2\pi C_1 z_2}\sqrt{\dfrac{7}{10}} = \dfrac{0.133}{C_1 z_2}$
3	(circuit diagram: C_1 series; z_2 shunt)	5	15.43	$\dfrac{1}{2\pi\sqrt{0.546}\,C_1 z_2} = \dfrac{0.215}{C_1 z_2}$

Table 2

No.	Diagram of feedback circuit	n	q2 ≪ 1	
			Minimum K	Generat. freq. f in cps
1	(circuit diagram: z_1, z_1, z_1 series; C_2, C_2, C_2 shunt)	3	29	$\dfrac{\sqrt{6}}{2\pi C_2 z_1} = \dfrac{11.39}{C_2 z_1}$
2	(circuit diagram: z_1 series; C_2 shunt)	4	18.38	$\dfrac{\sqrt{10}}{2\pi\sqrt{7}\,C_2 z_1} = \dfrac{0.19}{C_2 z_1}$
3	(circuit diagram: z_1 series; C_2 shunt)	5	15.43	$\dfrac{\sqrt{0.546}}{2\pi C_2 z_1} = \dfrac{0.118}{C_2 z_1}$

3. CONCLUSIONS

On the basis of our results we can draw the following conclusions:

1. Generation is possible if the number of cascade-connected, identical, half-T shaped quadripoles in the feedback circuit is more than two.

2. The greater the number of these quadripoles, the smaller the necessary amplification

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3. When we increase the number of cascade-connected links from fig.2a in
the feedback circuit, there is an increase in the value of C_1r_2 necessary to
obtain the given frequency (table 1).

4. When we increase the number of cascade-connected links from fig.2b in
the feedback circuit there is a decrease in the value of C_2r_1 necessary to obtain
the given frequency (table 2).

Conclusions 3 and 4 should be borne in mind when, in designing a generator,
we obtain inconvenient (very large or very small) values for the resistances and
capacities.

APPENDIX 1

The requirement for fulfilling the amplitude balance and phase balance so
as to develop generation in the closed linear system of fig.8, corresponds to
the equality:

$$\dot{U}_1 - \dot{U}_2. \qquad (I)$$

where $\dot{U}_1$ and $\dot{U}_2$ are the voltages at the input and output terminals (respectively)
of the quadripole that will be obtained if we divide fig.8 at any spot and load
its output terminals with a resistance equal to the repeat resistance of this
quadripole. If we divide fig.8 at points 1-1, we will obtain the quadripole of
fig.9. The repeat resistance of this quadripole equals infinity (since it
begins with the open input of the tube) and therefore its output terminals in
fig.9 are shown as being interrupted.

We know that for any quadripole the following equation is valid:

$$\dot{U}_1 = A_{11}\dot{U}_2 + A_{12}\dot{I}_2.$$

where A_{11} and A_{12} are the quadripole's parameters-coefficients.

The output terminals of the quadripole in fig.9 are interrupted, and there-
fore $I_2 = 0$ and the basic equation becomes:
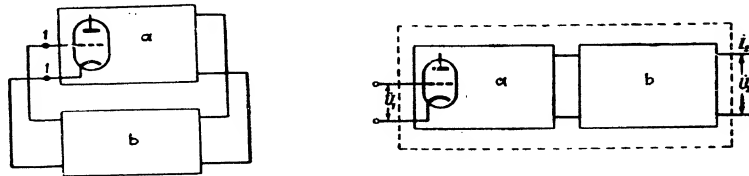
$$\dot{U}_1 - A_{11}\dot{U}_2.$$

Incorporating equality I, we obtain the following condition for the develop-

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ment of continuous oscillations in the closed linear system of fig.8:

$$A_{11} = 1$$

Here A_{11} is the parameter-coefficient of the quadripole from fig.9.

Fig.8 α(Amplifier) b(Feedback circuit)   **Fig.9**

This equality is the particular case of a correlation, advanced by E.V. Zelyakh in 1929, between the parameters-coefficients of a quadripole for determining the stability of a linear closed system and is only valid for the special case where the input resistance of a quadripole obtained from dividing a closed system is equal to infinity.

APPENDIX 2

By "bringing the square matrix $\|a\|$ to its diagonal form" (4) it is meant that this magnitude is presented in the following form:

$$\|a\| = \|x\| \cdot \|\lambda\| \cdot \|x\|^{-1};$$

Here $\|x\|$ is the diagonalizing matrix, $\|x\|^{-1}$ the reverse matrix to the diagonalizing one, and $\|\lambda\|$ the diagonal matrix.

If $\|a\|$ is a square matrix of the second order:

$$\|a\| = \begin{Vmatrix} a_{11} & a_{.2} \\ a_{21} & a_{.2} \end{Vmatrix};$$

then

$$\|x\| = \begin{Vmatrix} 1 & \dfrac{\lambda_2 - a_{22}}{a_{21}} \\ \dfrac{a_{21}}{\lambda_{.} - a_{22}} & 1 \end{Vmatrix},$$

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$$\|x\|^{-1} = \left\| \begin{array}{cc} 1 & \dfrac{a_{22} - \lambda_2}{a_{21}} \\ \dfrac{a_{21}}{a_{22} - \lambda_1} & 1 \end{array} \right\|.$$

and $\quad \|\lambda\| = \left\| \begin{array}{cc} \lambda_1 & 0 \\ 0 & \lambda_2 \end{array} \right\|.$

Here λ_1 and λ_2 are the matrix own values, the roots of equation:

$$\left| \begin{array}{cc} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{array} \right| = 0.$$

which is called the characteristic equation of matrix $\|a\|$.

Having solved this equation with regard to λ, we obtain:

$$\lambda_1 = \frac{a_{11} + a_{22}}{2} + \sqrt{\frac{(a_{11} + a_{22})^2}{4} - |a|}. \quad \text{(II)}$$

$$\lambda_2 = \frac{a_{11} + a_{22}}{2} - \sqrt{\frac{(a_{11} + a_{22})^2}{4} - |a|}. \quad \text{(III)}$$

where $|a|$ is the determinant of matrix $\|a\|$:

$$|a| = a_{11}a_{22} - a_{12}a_{21}.$$

As we know, in the case of a passive linear quadripole, $|a|$ equals 1.

For a matrix presented in its diagonal form, the operation of raising it to a power is highly simplified.

If we have a square matrix $\|a\|$, then:

$$\|a\|^n = \|x\| \cdot \|\lambda\|^n \cdot \|x\|^{-1}.$$

In the case where $\|a\|$ is a matrix of the second order, we obtain:

$$\|a\|^n = \|x\| \cdot \left\| \begin{array}{cc} \lambda_1^n & 0 \\ 0 & \lambda_2^n \end{array} \right\| \cdot \|x\|^{-1}.$$

After substituting the aforesaid matrixes $\|x\|$ and $\|x\|^{-1}$ and after multiplying, we obtain formula 8.

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FOOTNOTES

a) Patent No.48,581 having priority from November 22, 1953

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A.Z.FRADIN and V. A. OLENDSKY

ON THE ANTENNA EFFECT OF A SYMMETRICAL FEEDER

(A quantitative estimation is given of the antenna effect of a feeder. An analysis is presented of the antenna effect of a receiving, symmetrical, open feeder in the presence of asymmetry of the receiver's input. The study is based on the theory of assymetrical lines developed by A. A. Pistolkors, and on the multipole theory. Discussion is given of the nature of the relationship between the antenna effect and the magnitude of asymmetry, the parameters of the antenna-feeder system, the wave length and the parameters of the ground).

INTRODUCTION

One of the most important demands made of the feeder lines of receiving radio stations is the absence of antenna effect. A radical means for eliminating antenna effect is the application of shielded feeders, particularly coaxial cables. However, in the case of open feeders, the antenna effect can be eliminated or considerably weakened, firstly through the use of a four-conductor, alternating-phase (crossed) feeder line for eliminating the excitation of anti-phase emf, and secondly by symmetrizing the input of the receiver so as to eliminate the influence of cophasal emf's excited by the electromagnetic field in the feeder. Without discussing the effectiveness, in this application, of coaxial cables or other types of shielded feeders, we will dwell in this article on the effectiveness of the various means applied in open feeders.

We know that in the case of alternating-phase, four-conductor feeders in which there is a small distance between conductors, as in a standard receiving feeder, the anti-phase emf's in the feeder line are almost completely eliminated.

The influence of the receiver input's asymmetry on the antenna effect of an open feeder has received little attention in technical literature. A study of this question was first done by one of the authors of this article.

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In this article we expound a method for a general solution to problems dealing with the antenna effect of a feeder as caused by asymmetry of the receiver's input.

1. POSING THE PROBLEM

In Fig.1a we show the principal schematics of the circuit: antenna — feeder line — receiver input.

The principal schematics is shown in the projected form, i.e. one part of it (the tube and contour coil) is shown in terms of a circuit that is asymmetrical with regard to the ground, while the second part (antenna, feeder line and input elements of the receiver) in terms of a symmetrical circuit. In practice, it is impossible, in the second part of the schematic, to obtain complete symmetry with the ground. Some elements of asymmetry remain both in the antenna and feeder line, as well as in the input elements of the receiver. In this study we take into account only the asymmetry of the receiver's input elements; the antenna and feeder line are regarded as symmetrical.

For convenience of analysis let us replace, in conformity with the principle of reciprocity, the schematic of Fig.1a with an anological transmitting schematic, shown in Fig. 1b.

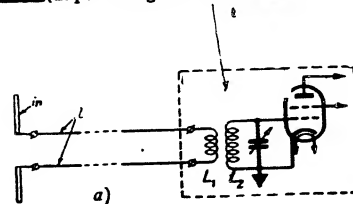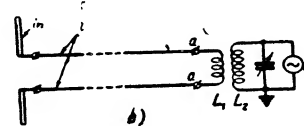<u>Fig. 1a</u> (Input stage of receiver)   <u>Fig. 1b</u>

To give the solution to the problem greater generality, let us replace, in conformity with the theory of multipoles (Bibl.1), the input receiver elements of Fig.1b with the tripole (three-terminal network) shown in Fig.2. Here Z_n, Z' and

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Z'' are the branch resistances, while E' and E'' are the emf at input terminals 1 and 2. Any receiver input schematic can be brought to the appearance of fig.2. Where the tripole is asymmetrical both in terms of resistances Z' and Z'' and in terms of the emf's E' and E'', there develops in the feeder together with the anti-phase current a cophase current which will be the cause of an appreciable antenna effect in the feeder.

Fig.2 (Lead 1, Lead 2, Feeder)
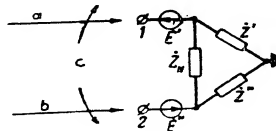
If we adopt the schematic of fig. 2, we must divide the problem into two parts, theoretical and practical. The first consists in determining the antenna effect of a symmetrical feeder line which at one end is loaded on the given symmetrical antenna and which on the other side is fed from a tripole of the fig.2 type. The second, practical, part of the problem consists in working out the most convenient way to measure, in the receiver, the equivalent parameters corresponding to the tripole of fig.2. In this study we expound only the solution to the theoretical part of the problem.

2. THE FEEDER'S RECEPTION FACTOR

With regard to the receiving system shown in fig. 1a, we will consider as the characteristic of the feeder's antenna effect the ratio of the voltage on the tube's grid produced by the electromotive forces excited by the arriving electromagnetic wave directly on the feeder, to the voltage on the tube's grid produced by the electromotive forces excited by the electromagnetic wave on the antenna's leads. To make the problem more definite, let us assume that the electromagnetic waves

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that excite both the antenna and the feeder have an identical field intensity and
arrive from the maximum reception directions of the antenna and feeder respective-
ly. We will call this ratio the feeder reception factor (f.r.f) and designate it
by the letter N.

For the transmitting diagram of fig.1b, we will define the f.r.f. as the
ratio of the emission field intensity of the feeder in the main direction to the
emission field intensity of the antenna in the main direction.

Taking into consideration that the feeder's emission is basically determined by
the cophasal wave on it, and the antenna's emission by the anti-phase wave, we
can determine the f.r.f. as follows:

$$N = \frac{\sqrt{P_c \cdot G_\phi}}{\sqrt{P_\Pi \cdot G_A}} = \frac{I_{cl}}{I_{\Pi l}}\sqrt{\frac{R_{cl}\cdot G_\phi}{R_{\Pi l}\cdot G_A}} = \frac{I_{cl}}{I_{\Pi l}}\sqrt{\frac{120}{R_{\Pi l}\cdot G_A}}\cdot\frac{\pi h_{\partial\phi}}{\lambda}, \quad (1)$$

where: P_c is the power of the cophasal wave current;

G_ϕ is the feeder's amplification factor;

P_Π is the power of the anti-phase wave currents;

G_A is the antenna's amplification factor;

I_{cl} is the cophasal current in the feeder line leads at points a-a (fig.1b);

$I_{\Pi l}$ is the anti-phase current in the feeder line leads at points a-a;

R_{cl} is the effective resistance component of the antenna-feeder system at
points a-a for the cophasal current;

$R_{\Pi l}$ is the same as above for the anti-phase current;

$h_{\partial\phi}$ is the acting height of the feeder line;

λ is the wave length.

3. THE ELECTRICAL PARAMETERS OF THE FEEDER LINE

The values of the currents and resistances entering into formula 1 can be
found by using methods of the long line theory. The theory of long lines, as we
know, is based on differential equations in which the linear parameters of the lines
are used as the multipliers. Let us speak for a moment about the characteristics
of these parameters in the case of feeder lines. If we were to limit ourselves

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to studying only the anti-phase wave, the feeder line might be regarded as a long line situated over an ideally conductive ground. The linear parameters of such a symmetrical line are: the inductance L_o, the capacity C_o, the effective resistances r_o of the leads, the mutual inductance M_{12} and the mutual capacity C_{12} between leads.

In the given case, since our examination includes, together with the anti-phasal wave, the cophasal wave, in addition to the aforesaid parameters we must also incorporate parameters introduced by the semi-conductive ground: the linear effective resistances r_{g0} and r_{g12} and the introduced inductances L_{g0} and M_{g12}, as well as the linear emission resistances of the cophasal currents $r_{\Sigma 0}$ and $r_{\Sigma 12}$. We will ignore the effective leakage of the leads in this case.

We can gain a notion of the order of magnitudes of the aforesaid parameters from the table given below. In the case of an ideally conductive ground, the parameters were calculated by the method of potential coefficients (2), while the parameters bound up with the influence of a semi-conductive ground were calculated by the method expounded in Bibl. 3 and 4. The data given in the table refers to a symmetrical two-lead feeder suspended over moist earth ($\sigma = 10^{-13}$ CGSM : $\varepsilon = 10$) and over dry earth ($\sigma = 10^{-14}$ CGSM : $\varepsilon = 5$) at a height h equals 4m. The distance between the feeder leads was adopted as D equals 35mm and the lead radius as $R \approx$ 6.1mm. This kind of feeder, in terms of wave resistance, is equivalent to the standard quadripole receiver feeder having a lead radius of r equals 0.75 mm and a distance between leads of d equals 35mm.

As a supplement to the data in the table, fig. 3 and 4 show curves of the relationship of r_{g0} and L_{g0} to the wave length in the range from 15 to 100 m, the curves being drawn up for different suspension heights of the feeder over damp earth.

A notion as to the order of magnitudes of the feeder line's emission resistances

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can be gotten from fig. 5, in which we give the values of emission resistances $r_{\Sigma O}$ as calculated for a harmonic lead suspended at different heights h over an ideally conductive ground. The values of the emission resistances are given in relation to the harmonic number n.

Fig. 3.
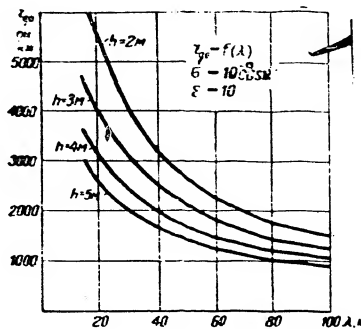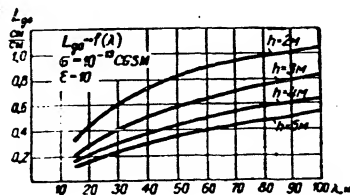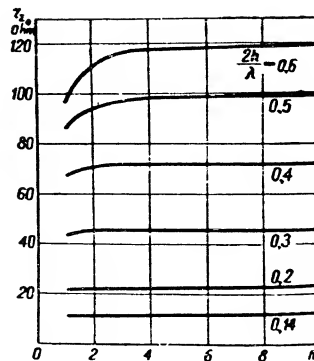

Fig. 4.

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Fig.5.

4. THE ANTENNA EFFECT IN A FEEDER

The feeder reception factor, formula 1, consists of three multipliers, the simplest of which is the multiplier $\sqrt{\dfrac{120}{R_{\eta l} \cdot G_A}}$. The input resistance of the antenna-feeder system, $R_{\eta l}$, usually has a value equal to the wave resistance of the feeder, W_ϕ equals 200 cm. The amplification factor G_A of modern antennas has been rather carefully studied. Depending on the type of antenna, its magnitude can have values of from a few units to several hundreds.

Table – see next page .

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Table. Data on electrical parameters of a standard receiving feeder.

Kinds of soil	λ	Lead parameters in the case of an ideally conductive ground					Lead parameters conditioned by the influence of a semi-conductive ground			
OGS units	m	$c_0\left(\frac{cm}{ocm}\right)$	$c_{12}\left(\frac{cm}{cm}\right)$	$r_0\left(\frac{ohm}{ocm}\right)$	$L_0\left(\frac{cm}{ocm}\right)$	$M_{12}\left(\frac{cm}{cm}\right)$	$L_{g0}\left(\frac{cm}{ocm}\right)$	$r_{g0}\left(\frac{ohm}{km}\right)$	$M_{g12}\left(\frac{cm}{cm}\right)$	$r_{g12}\left(\frac{ohm}{km}\right)$
Moist soil	20	0.163	0.123	110	14.3	10.8	0.206	3136	0.206	3136
$\sigma = 10^{-13}$	40	0.163	0.123	77.5	14.3	10.8	0.376	2014	0.376	2014
$\varepsilon = 10$	60	0.163	0.123	64.0	14.3	10.8	0.490	1509	0.490	1509
Dry soil	20	0.163	0.123	110	14.3	10.8	0.224	6610	0.224	6610
$\sigma = 10^{-14}$	40	0.163	0.123	77.5	14.3	10.8	0.712	4760	0.712	4760
$\varepsilon = 5$	60	0.163	0.123	64.0	14.3	10.8	1.052	2910	1.052	2910

The multiplier $\pi \frac{h_{2e}}{\lambda}$ for the feeder situated over semi-conductive soil has a complex relationship to the wave length λ, the height of the feeder over the ground's surface and the parameters of the soil.

An analysis, which we omit because of its length, shows that in the high-frequency range, at practically applied feeder heights h above the ground, the acting height of the feeder h_{2e} depends, basically, only on the suspension height of the feeder line and its conditions, and varies approximately from h, for lines with large attenuation, to 4h, for lines with extremely small attenuation, working under the conditions of a moving wave. This also gives us a notion as to the magnitude of $\pi \frac{h_{2e}}{\lambda}$.

The expression for the multiplier $\dot{I}_{o1}/\dot{I}_{\pi_1}$ has the following form (see the appendix);

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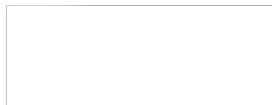
$$\frac{\dot{I}_{cl}}{\dot{I}_{\pi_1}} = \frac{\dot{A}_E \left(1 + \frac{\dot{Z}_{A\phi\Pi}}{\dot{Z}_{sr}}\right) - \dot{A}_s}{2\dot{M} + \frac{1}{2}(1 - \dot{A}_E \dot{A}_Z) + \frac{\dot{Z}_{A\phi c}}{\dot{Z}_{sx}}}, \qquad (2)$$

where $\dot{A}_E$ equals $\frac{\dot{E}' + \dot{E}''}{\dot{E}' - \dot{E}''}$ is the asymmetry coefficient in terms of the emf's;

$\dot{A}_Z$ equals $\frac{\dot{z}' + \dot{z}''}{\dot{z}' - \dot{z}''}$ is the asymmetry coefficient in terms of the resistances;

$\dot{M}$ equals $\frac{\dot{z}' \cdot \dot{z}''}{Z_{\pi}(\dot{z}' + \dot{z}')}$ is the ratio of the middle branch conductance to the total
conductance of the side branches of the tripole (fig.2);

$\dot{Z}_{A\phi\Pi}$, $\dot{Z}_{A\phi c}$ is the input resistance of the antenna-feeder system in terms of
the anti-phasal and cophasal currents respectively;

$\dot{Z}_{ax}$ is the receiver's input resistance.

In formula 2, the basic parameters determining the degree of asymmetry are A_E
and A_Z. Their adjustment permits us to symmetrize the receiver's input and essen-
tially to bring ratio $\dot{I}_{cl}/I_{\pi1}$ to zero. The receiver's input is only completely
symmetrical in the case where $\dot{A}_E$ equals 0 and $\dot{A}_Z$ equals 0 simultaneously. Ratio
$\dot{I}_{cl}/I_{\pi1}$ equals 0 not only where $\dot{A}_E$ equals 0 and $\dot{A}_Z$ equals 0, but also where
$\dot{A}_E(1 \neq \frac{Z_{A\phi\Pi}}{Z_{ax}})$ equals $\dot{A}_Z$, i.e. where there is mutual compensation of asymmetry.

Ratio $\dot{I}_{cl}/I_{\pi1}$ also depends on $\dot{Z}_{A\phi\Pi}$, $\dot{Z}_{A\phi c}$, $\dot{Z}_{ax}$ and M. However, the possibility
of changing $\dot{I}_{cl}/I_{\pi1}$ by specially choosing these magnitudes is highly restricted.
Indeed, $\dot{Z}_{A\phi\Pi}$ and Z_{ax}, for purposes of the maximum effectiveness of the receiving
system, are usually made equal to the wave resistance of the feeder. The magni-
tude of M cannot increase unlimitedly since the parasite leakages $\dot{Z}'$ and $\dot{Z}''$ princi-
pally have final values. With regard to the resistance $\dot{Z}_{A\phi c}$, for the given con-
crete antenna-feeder system its magnitude is completely determined and cannot be
adjusted.

Expression 2 together with formula 1 allows us to find the magnitude of the
antenna effect of a symmetrical feeder developed owing to the asymmetry of the
receiver's input.

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CONCLUSION

On the basis of our study, we can draw the following conclusions:

1. It is convenient to evaluate the magnitude of the feeder's antenna effect by means of the so-called feeder reception factor, which, when replacing the receiving schematic with the equivalent transmitting schematic, is defined as the ratio of the emission field intensity of the feeder in the main direction to the emission field intensity of the antenna in the main direction.

2. The f.r.f. is a complex function of the parameters and characteristics of the antenna-feeder system and of the receiver's input.

3. The f.r.f. is most evident as a function of the asymmetry coefficients of the receiver's input, the antenna's amplification factor, the feeder's acting height and the feeder's suspension height over the ground's surface.

4. The asymmetry of the receiver's input should be characterized by two coefficients, which are called the asymmetry coefficient in terms of the resistances and the asymmetry coefficient in terms of the emf's.

5. The method of solving the problem, as proposed in this study, can easily be generalized to the case of a transmitting antenna-feeder system.

APPENDIX

DERIVATION OF FORMULA FOR COPHASAL AND ANTI-PHASAL CURRENT RATIO IN FEEDER LINES

Based on the fundamental theses of the theory of asymmetrical, electrically connected lines developed by A. A. Pistolkors (2), we can present the equations for the currents and potentials in the leads of a two-lead line (fig.6) in the following form:

$$\frac{d\dot{V}_1}{dx} = \dot{Z}_1 \dot{I}_1 + \dot{Z}_{12} \dot{I}_2; \quad \frac{d\dot{V}_2}{dx} = \dot{Z}_2 \dot{I}_2 + \dot{Z}_{12} \dot{I}_1$$
$$\frac{d\dot{I}_1}{dx} = i(b_1 \dot{V}_1 - b_{12} \dot{V}_2); \quad \frac{d\dot{I}_2}{dx} = i(b_2 \dot{V}_2 - b_{12} \dot{V}_1) \tag{1}$$

where $\dot{V}_1$ and $\dot{V}_2$ are the potentials of leads 1 and 2 relative to the ground;

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where $\dot{I}_1$ and $\dot{I}_2$ are the currents in leads 1 and 2;

$\dot{Z}_1$, $\dot{Z}_2$, $\dot{Z}_{12}$, b_1, b_2 and b_{12} are the complex parameters of the line, equalling:
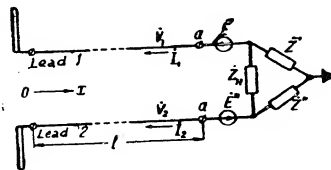
$$\dot{Z}_1 = (r_1 + r_{\Sigma 1} + r_{g1}) + i\,\omega\,(L_1 + L_{g1});$$

$$\dot{Z}_2 = (r_2 + r_{\Sigma 2} + r_{g2}) + i\,\omega\,(L_2 + L_{g2});$$

$$\dot{Z}_{12} = (r_{\Sigma 12} + r_{g12}) + i\,\omega\,(M_{12} + M_{g12});$$

$$b_1 = \omega C_1; \quad b_2 = \omega C_2; \quad b_{12} = \omega C_{12}$$

Fig. 6.

Taking into consideration that in this study we are examining symmetrical feeder lines, i.e. lines in which the leads have identical diameters and are found at an identical height above the ground, it is possible to assume:

$$\dot{Z}_1 = \dot{Z}_2 = \dot{Z}_0 \quad \text{and } b_1 = b_2 = b_0. \tag{2}$$

Introducing designations:

$\dot{V}_1 - \dot{V}_2 = U_\pi$ is the voltage between the feeder's leads (the anti-phase voltage);

$\frac{1}{2}(\dot{V}_2 + \dot{V}_2) = \dot{U}_0$ is the feeder's potential relative to the ground;

$\frac{1}{2}(\dot{I}_2 - \dot{I}_1) = \dot{I}$ is the anti-phasal current in each lead of the feeder;

$(\dot{I}_2 + \dot{I}_1) = \dot{I}_c$ is the total cophasal current in the feeder,

and, taking into account correlation 2, we can give equation 1 the following appearance:

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$$\frac{d\dot{J}_{\Pi}}{dx} = 2(\dot{Z}_0 - \dot{Z}_{12})\dot{I}_{\Pi} \Bigg\} \quad (4) \qquad\qquad \frac{d\dot{U}_c}{dx} = \frac{1}{2}(\dot{Z}_0 + \dot{Z}_{12})\dot{I}_c \Bigg\} \quad (5)$$

$$\frac{d\dot{I}_{\Pi}}{dx} = i(b_0 + b_{12})\dot{U} \qquad\qquad \frac{d\dot{I}_c}{dx} = i\,2(b_0 - b_{12})\dot{U}_c$$

Equations 4 represent the customary equations for a two-lead line (equations for the anti-phasal wave of the current and voltage), while equations 5 are the usual equations for a single-lead line having a return current path through the ground (equation for the cophasal wave of the current and voltage).

Equations 4 and 5, as we know, have the following solutions:

$$\dot{U}_{\Pi} = \dot{A}_{\Pi}\,\text{ch}\,\dot{\gamma}_{\Pi}\,x + \dot{B}_{\Pi}\,\text{sh}\,\dot{\gamma}_{\Pi}\,x \Bigg\} \quad (6) \qquad \dot{U}_c = \dot{A}_c\,\text{ch}\,\dot{\gamma}_c\,x + \dot{B}_c\,\text{sh}\,\dot{\gamma}_c\,x \Bigg\} \quad (7)$$

$$\dot{I}_{\Pi} = \frac{1}{\dot{\rho}_{\Pi}}(A_{\Pi}\,\text{sh}\,\dot{\gamma}_{\Pi}\,x + B_{\Pi}\,\text{ch}\,\dot{\gamma}_{\Pi}\,x) \qquad \dot{I}_c = \frac{2}{\dot{\rho}_c}(A_c\,\text{sh}\,\dot{\gamma}_c\,x + B_c\,\text{ch}\,\dot{\gamma}_c\,x)$$

where $\dot{\gamma}_{\Pi} = \sqrt{i(\dot{Z}_0 + \dot{Z}_{12})(b_0 - b_{12})}$ is the propagation constant of the anti-phasal current;

$\dot{\rho}_{\Pi} = 2\sqrt{i\dfrac{\dot{Z}_{12} - \dot{Z}_0}{b_0 + b_{12}}}$ is the characteristic resistance of the line for the anti-phasal current;

$\dot{\gamma}_c = \sqrt{i(\dot{Z}_0 + \dot{Z}_{12})(b_0 - b_{12})}$ is the propagation constant of the cophasal current;

$\dot{\rho}_c = \sqrt{i\dfrac{\dot{Z}_0 + \dot{Z}_{12}}{b_{12} - b_0}}$ is the characteristic resistance of the line for the cophasal current;

$\dot{A}_{\Pi};\ \dot{B}_{\Pi};\ \dot{A}_c\,\text{and}\,\dot{B}_c$ are the integration constants determined from the problem's boundary conditions.

At the connection point of the feeder line to the antenna (where x equals 0) we have the following correlations between voltages $\dot{U}_{\Pi_0}$, $\dot{U}_{C_0}$ and currents I_{Π_0}, I_{C_0} :

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$$\left.\begin{array}{c} \dot{U}_{\Pi_0} - \dot{I}_{\Pi_0} \dot{Z}_{A\Pi} \\ U_{C0} - I_{C0} \dfrac{1}{2} Z_{AC} \end{array}\right\} \tag{8}$$

where $Z_{A\Pi}$ is the antenna's input resistance to the anti-phasal current wave;

Z_{AC} is the antenna's input resistance to the cophasal current wave.

Introducing designations $\dot{Z}_{A\Pi} = \dot{\rho}_\Pi \, \text{th} \, \theta_{A\Pi}$ and $\dot{Z}_{AC} = \dot{\rho}_C \, \text{th} \, \theta_{AC}$ and applying boundary conditions 8 to equations 6 and 7, we obtain, after some simple transformations:

$$\left.\begin{array}{c} \dot{U}_\Pi = \dot{I}_{\Pi_0} \, \dfrac{\dot{\rho}_{\Pi}}{\text{ch} \, \theta_{A\Pi}} \, \text{sh}\,(\theta_{A\Pi} + \dot{\tau}_\Pi \, x) \\[2mm] \dot{I}_\Pi - \dot{I}_{\Pi_0} \, \dfrac{1}{\text{ch} \, \theta_{A\Pi}} \, \text{ch}\,(\theta_{A\Pi} + \dot{\tau}_\Pi \, x) \end{array}\right\} \tag{9}$$

$$\left.\begin{array}{c} \dot{U}_C = \dot{I}_{C0} \, \dfrac{\dot{\rho}_C}{2\,\text{ch} \, \theta_{AC}} \, \text{sh}\,(\theta_{AC} + \dot{\tau}_C \, x) \\[2mm] \dot{I}_C = \dot{I}_{C0} \, \dfrac{1}{\text{ch} \, \theta_{AC}} \, \text{ch}\,(\theta_{AC} + \dot{\tau}_C \, x) \end{array}\right\} \tag{10}$$

As we see from the previous tables

$$r_0 \ll r_{g0}; \quad L_{g0} \ll L_0; \quad M_{g12} \ll M_{12}; \quad r_{g0} \approx r_{g12} \text{ и } L_{g0} \approx M_{g12}. \tag{11}$$

In addition, for feeder lines having a small distance between leads in comparison with the wave length, we can assume that;

$$r_{\Sigma_0} \approx r_{\Sigma_{12}}. \tag{12}$$

Incorporating formulas 11 and 12, we can assume with a sufficient degree of precision that:

$$\dot{\tau}_\Pi - i\,\dfrac{2\pi}{\lambda} - i\,m; \qquad \dot{\rho}_\Pi - W_\Phi, \tag{13}$$

where λ is the wave length, W_Φ the wave resistance of the feeder for the antiphasal current;

$$\dot{\rho}_C = W_C; \quad \dot{\gamma}_C = \dfrac{r'_{\Sigma_0} + r_{g0}}{W_C} + im = \beta_C + im \tag{14}$$

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where W_C is the wave resistance of the feeder for the cophasal current;

β_C is the attenuation factor of the feeder for the cophasal wave.

The wave resistance W_C is calculated from the formulas:

$$\overline{W}_C = 120 \ln \frac{2h}{\sqrt{RD}} \text{ for two-lead feeders}$$

and

$$W_C = \frac{W_\Phi}{2} + 120 \ln \frac{2h}{d} \text{ for four-lead feeders.}$$

The designations of h, R, D and d correspond to those indicated in part 3.

The values of the attenuation factor β_C for the cophasal current of a standard four-lead receiving feeder are given in graph form in fig.7. The graph applies to a moist earth ($0 = 10^{-13}$ CGSM; $\varepsilon = 10$) and to case where the linear resistance of the feeder's emission $r_{\Sigma 0}$ can be ignored in relation to the effective linear resistance r_{g0}, introduced into the feeder's lead by a semi-conductive ground. We note that in the high-frequency range, this is always valid if the length of the feeder $l > 150$-200 m.
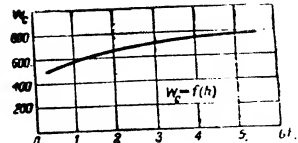
Fig.7.
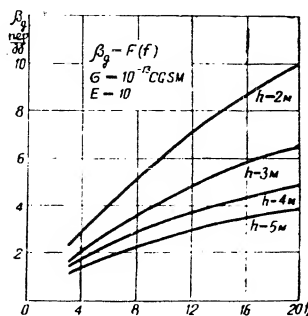
In fig.8 we give a graph showing the relationship between the wave resistance

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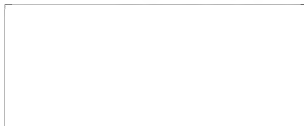
W_C of a standard four-lead receiving feeder and the suspension height of the feeder over the ground.

Fig.8.

Taking into account formulas 13 and 14, we can bring equations 9 and 10 into the following form:

$$U_\pi = \dot{I}_{\pi 0}\ \frac{W_\phi}{\text{ch}\,\Theta_{A\pi}}\ sh\,(\phi_{A\pi} + im\,x)$$
$$\dot{I}_\pi = \dot{I}_{\pi 0}\ \frac{1}{\text{ch}\,\Theta_{A\pi}}\ \text{ch}\,(\Theta_{A\pi} + i\,m\,x) \tag{15}$$

$$U_C = \dot{I}_{C0}\,\frac{W_C}{2\,\text{ch}\,\Theta_{AC}}\ sh\,(\Theta_{AC} + i_C\,x)$$
$$i_C = \dot{I}_{C0}\,\frac{1}{\text{ch}\,\Theta_{AC}}\ \text{ch}\,(\Theta_{AC} + i_C\,x) \tag{16}$$

There now remains to determine the coefficients I_{C0} and $I_{\pi 0}$ from the boundary conditions on the other end of the feeder line where x equals l. For this let us turn to fig.6. In view of the asymmetry of the tripole we cannot present the boundary conditions on this end in the form of a simple correlation between the anti-phasal current $\dot{I}_{\pi 1}$ and the cophasal voltage U_{C_1}, as we did at the antenna end of the feeder. In this case, the anti-phasal current will also depend on the cophasal voltage, while the cophasal current will depend on the anti-phasal voltage.

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In view of the above, the most convenient thing to do is to express I_{C1} and $\dot{I}_{\Pi 1}$ through the values of $\dot{V}_{1l}$ and $\dot{V}_{2l}$, which are bound up with $\dot{U}_{C1}$ and $\dot{U}_{\Pi 1}$ by correlations 3.

The boundary conditions where x equals l can be expressed through $\dot{V}_{1l}$ and $\dot{V}_{2l}$ in the following form:

$$\left.\begin{aligned}
\dot{I}_{\Pi l} &= \frac{(\dot{E}' - \dot{V}_{1l}) - (\dot{E}'' - \dot{V}_2 l)}{Z_\varkappa} + \frac{1}{2}\left(\frac{\dot{E}' - \dot{V}_{1l}}{\dot{z}'} - \frac{\dot{E}'' - \dot{V}_{2l}}{\dot{z}''}\right) \\
\dot{I}_{Cl} &= \frac{\dot{E}' - \dot{V}_{1l}}{\dot{z}'} + \frac{\dot{E}'' - \dot{V}_{2l}}{\dot{z}''}.
\end{aligned}\right\} \quad (17)$$

Expressing $\dot{V}_{1l}$ and $\dot{V}_{2l}$ through $\dot{U}_{C1}$ and $\dot{U}_{\Pi 1}$ according to formula 3, we can transform formula 17 into the following form:

$$\left.\begin{aligned}
\dot{I}_{\Pi l} &= \frac{E' - E''}{Z_\varkappa} + \frac{1}{2}\left(\frac{\dot{E}'}{\dot{z}'} - \frac{\dot{E}''}{\dot{z}''}\right) - \dot{U}_{\Pi l}\left[\frac{1}{\dot{z}_\varkappa} + \frac{1}{4}\left(\frac{1}{\dot{z}'} + \frac{1}{\dot{z}''}\right)\right] - \\
&\qquad - \frac{\dot{U}_{Cl}}{2}\left(\frac{1}{\dot{z}'} - \frac{1}{\dot{z}''}\right) \\
\dot{I}_{Cl} &= \frac{\dot{E}'}{\dot{z}'} + \frac{\dot{E}''}{\dot{z}''} - \dot{U}_{Cl}\left(\frac{1}{\dot{z}'} + \frac{1}{\dot{z}''}\right) - \frac{\dot{U}_{\Pi l}}{2}\left(\frac{1}{\dot{z}'} - \frac{1}{\dot{z}''}\right)
\end{aligned}\right\} \quad (18)$$

Substituting into formulas 15 and 16 the values of $\dot{I}_{\Pi 1}$ and $\dot{I}_{C1}$ from formula (18), we obtain a system made up of two equations with two unknowns $\dot{I}_{\Pi 0}$ and $\dot{I}_{C0}$, from which it follows that:

$$\frac{\dot{I}_{\Pi 0}}{\mathrm{ch}\,\Theta_{A\Pi}} = $$

$$-\frac{\left[\frac{\dot{E}' - \dot{E}''}{\dot{Z}_\varkappa} + \frac{1}{2}\left(\frac{\dot{E}'}{\dot{z}'} - \frac{\dot{E}''}{\dot{z}''}\right)\right]\left[\mathrm{ch}\,(\Theta_{AC} + \dot{\tau}_{Cl}) + \frac{W_C\,\dot{Y}_{(+)}}{2}\,\mathrm{sh}\,(\Theta_{AC} + \dot{\tau}_{Cl})\right]}{\Delta} - \quad (19)$$

$$-\frac{\left(\frac{\dot{E}'}{\dot{z}'} + \frac{\dot{E}''}{\dot{z}''}\right)\frac{W_C\,\dot{Y}_{(-)}}{4}\,\mathrm{sh}\,(\Theta_{AC} + \dot{\tau}_{Cl})}{\Delta}$$

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$$\frac{\dot{I}_{C0}}{ch\,\Theta_{AC}} =$$

$$= \frac{\left(\dfrac{\dot{E}'}{\dot{Z}'} + \dfrac{\dot{B}''}{\dot{Z}''}\right)\left[ch\,(\Theta_{A\Pi} + i\,ml) + \dfrac{W_\phi}{4\dot{Z}_\varkappa}(4 + \dot{Z}_\varkappa Y_{(+)})\,sh\,(\Theta_{A\Pi} + i\,ml)\right]}{\Delta} - \qquad (20)$$

$$- \frac{\left[\dfrac{\dot{E}' - \dot{E}''}{\dot{Z}_\varkappa} + \dfrac{1}{2}\left(\dfrac{\dot{E}'}{\dot{Z}'} - \dfrac{\dot{E}''}{\dot{Z}''}\right)\right]\dfrac{W_\phi \cdot \dot{Y}_{(-)}}{2}\,sh\,(\Theta_{A\Pi} + lml)}{\Delta},$$

where Δ is the system's determinant.

$$\dot{Y}_{(+)} = \frac{1}{\dot{Z}'} + \frac{1}{\dot{Z}''}; \qquad \dot{Y}_{(-)} = \frac{1}{\dot{Z}'} - \frac{1}{\dot{Z}''}$$

Substituting into formulas 15 and 16 x equals 1 and the values of $\dot{I}_{\Pi 0}$ and $\dot{I}_{CO}$, from formulas (19) and (20) after some simple transformation we can obtain the following expression for $\dot{I}_{c1}/\dot{I}_{\Pi 1}$:

$$\frac{\dot{I}_{cl}}{\dot{I}_m} =$$

$$= \frac{\dfrac{\dot{E}' + \dot{E}''}{\dot{E}' - \dot{E}''} - \dfrac{\dot{Z}' - \dot{Z}''}{\dot{Z}' + \dot{Z}''} + \dfrac{\dot{E}' + \dot{E}''}{\dot{E}' - \dot{E}''}\,\dfrac{W_\Phi}{\dot{Z}_{Bx}}\,th\,(\Theta_{A\Pi} + iml)}{2\,\dfrac{\dot{Z}'\dot{Z}''}{\dot{Z}_{\Pi}\,(\dot{Z}' + \dot{Z}'')} + \dfrac{1}{2}\left(1 - \dfrac{\dot{E}' + \dot{E}''}{\dot{E}' - \dot{E}''} \cdot \dfrac{\dot{Z}' - \dot{Z}''}{\dot{Z}' + \dot{Z}''}\right) + \dfrac{W_c}{\dot{Z}_{Bx}}\,th\,(\Theta AC + \dot{\gamma}_c l)} \qquad (21)$$

where $\dot{Z}_{Bx}$ is the input resistance of the receiver, equal to $\dot{Z}_{Bx} = \dfrac{\dot{Z}_\varkappa(\dot{Z}' + \dot{Z}'')}{\dot{Z}' + \dot{Z}'' + \dot{Z}_\varkappa}$

Designating the resistance of the antenna-feeder system to the anti-phasal current through $\dot{Z}_{A\phi\Pi} = W_\phi\,th(\Theta_{A\Pi} + lml)$ and to the cophasal current through $\dot{Z}_{A\phi 0} = W_e\,th(\Theta_{AC} + \dot{\jmath}_C l)$, and also, designating the ratio of the total of emf's to their difference through $\dot{A}_E$ equals $\dfrac{\dot{E}' + \dot{E}''}{\dot{E}' - \dot{E}''}$, the ratio of the difference of the side branch resistances to their total through $\dot{A}_z$ equals $\dfrac{\dot{Z}' - \dot{Z}''}{\dot{Z}' + \dot{Z}''}$ and the magnitude $\dfrac{\dot{Z}'\dot{Z}''}{\dot{Z}_\varkappa(\dot{Z}' + \dot{Z}'')}$ through M, expression 21 can be written in the following form:

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$$\frac{\dot{I}_{c\,l}}{\dot{I}_{\Pi\,l}} = \frac{\dot{A}_E \quad 1 + \dfrac{\dot{Z}_{A\varphi\Pi}}{\dot{Z}_{Bx}} \quad - A_Z}{2\dot{M} + \dfrac{1}{2} \quad 1 - \dot{A}_E\dot{A}_Z \quad + \dfrac{\dot{Z}_{A\varphi c}}{\dot{Z}_{Bx}}} \qquad (22)$$

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RECTIFIERS WITH ELECTRONIC STABILIZATION

(A technique is suggested for the technical calculation of stabilization limits for rectifiers with electronic stabilization in terms of the given variations of the line voltage, the load current and the adjustment limits of the stabilized voltage.)

1. INTRODUCTION

Rectifiers with electronic stabilization are used very frequently at present. However, literature on the subject does not give sufficient information on calculating the characteristics of this device in terms of the given conditions.

Usually one analyzes the device's stabilization factor, which characterizes the variations of voltage or current at the device's input under conditions of pre-assigned variations of the line voltage or load current.

However, of no less interest is the matter of the stabilization limits, i.e. the extreme values of line voltage load current at which the device operates normally, as characterized by the given stabilization factor. In a rectifier with electronic stabilization it is easy to adjust the stabilized voltage. The limits of possible adjustment are bound up with the stabilization limits.

Calculation of stabilized rectifiers should be divided into two steps: the first, calculation of the stabilization limits, and the second, calculation of the stabilization factor. This article is devoted to the first calculation step, which involves the matters enumerated above.

Let us examine a rectifier built on kenotrons ($Л_1$, $П_2$) with a capacity input (Fig.1), in its most customary form.

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Fig.1.
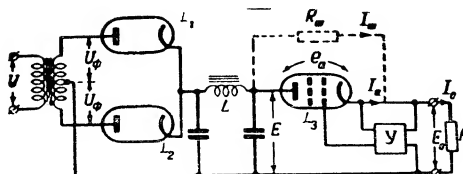
Tube L_3 is a regulating tube which plays the part of an automatically varying resistor absorbing the surplus rectified voltage E that augments both when the line voltage U rises and when the load resistance R increases. Y is a d.c. amplifier. We assume as known the performance of L_3, and the permissible power of dispersion on the anode. To obtain a greater stabilization factor, as the regulating tube we can use ray tubes or pentodes (with a separate source of voltage for the screen grid). When we have a triode connection of these tubes, we must take account of possible overheating in the screen grid, owing to which the permissible current magnitude decreases.

It is profitable to use tubes having minimum plate voltage at the assigned current through the lamp and a grid voltage of zero. For example, in a double triode 6H5C where e_c equals 0 and e_a equals 30 V, the anode current i_a equals 0.2 amps. If the load current surpasses the magnitude of the permissible anode current of the regulating tube, one hooks up several tubes in parallel or shunts the regulating tube with an effective resistor R_w (fig.1). Sometimes one hooks up an incandescent lamp as the shunt R_w.

2. CHANGE OF RECTIFIED VOLTAGE E DURING CHANGE OF LINE VOLTAGE

It is usually considered that the rectified non-stabilized voltage E in a stabilized kenotron rectifier (fig.1) augments proportionally to the line voltage (Bibl.1). This assumption is highly inaccurate since during the change of the voltage amplitude of the phase winding $U_{\phi m}$, the rectified current I_0 remains constant

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(since stabilization occurs), while the cut-off angle θ of the current through the kenotron varies.

For two values of $U_{\phi m}$ and θ we can therefore write out the following correlation:

$$I_0 = \frac{2}{\pi}\,\frac{U_{\phi m1}}{r}\,(\sin\Theta_1 - \Theta_1\cos\Theta_1) = \frac{2}{\pi}\,\frac{U_{m\phi_2}}{r}\,(\sin\Theta_2 - \Theta_2\cos\Theta_2),$$

where r is the effective phase resistance (the internal resistance of the kenotron and of the transformer's phase winding).

Hence:

$$\frac{\sin\Theta_1 - \Theta_1\cos\Theta_1}{\sin\Theta_2 - \Theta_2\cos\Theta_2} = \frac{U_{\phi m2}}{U_{\phi m1}}, \tag{1}$$

i.e. change in the cut-off angle is determined solely by change in the voltage of the phase winding.

On the other hand, the rectified voltage in these two cases equals:
$$E_1 = U_{\phi m1}\cos\Theta_1 \text{ и } E_2 = U_{\phi m2}\cos\Theta_2.$$

Let us designate:

$$\alpha = E_2/E_1 \tag{2}$$

and

$$s = U_{\phi m2}/U_{\phi m1}. \tag{3}$$

With sufficient accuracy for a technical calculation we can assume that the coefficient s characterizes the line voltage change and equals:

$$s = U_2/U_1. \tag{4}$$

The coefficient α characterizes the augmentation of the non-stabilized rectified voltage E during augmentation of the line voltage and with an unvaried load.

It is evident that:

$$\alpha = s\,\frac{\cos\Theta_2}{\cos\Theta_1}. \tag{5}$$

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Considering formula 1, we can say that the change of the rectified voltage α is the simple function of s and of the initial value of the cut-off angle θ (for example, θ_1), regarded as a parameter. The relationships $\alpha = F(s)$ for angles θ from 30° to 60°, the most frequently applied in kenotron rectifiers, are given in fig.2 for $s < 1$ and in fig.3 for $s > 1$.
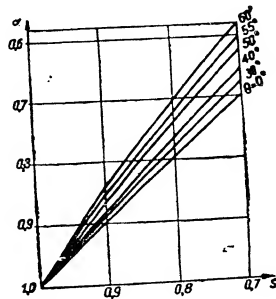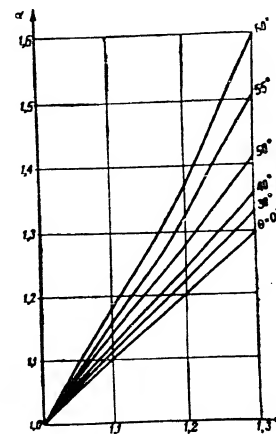
Fig.2.

Fig.3.

These graphs were calculated as follows. We constructed an auxiliary curve $P(\theta)$ equals $\sin\theta - \theta\cos\theta$. According to the selected value of θ_1 we determined the magnitude of $P(\theta_1)$. This magnitude was multiplied by s, from which we found the value of $P(\theta_2)$ (from formula 1). From the same auxiliary curve we determined the value of θ_2, after which the coefficient α, corresponding to the given s, was found from formula 5.

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As follows from the graphs, α differs the greater from s, the larger the cut-off angle, i.e. the smaller the load resistance of the kenotron rectifier. It should be noted that a comparatively small error in determining α results in a far larger error in determining the voltage on the regulating tube, and consequently, a greater error in determining the permissible (from the point of view of losses at anode L_3) augmentation of voltage in the network.

In a hot-cathode rectifier, the cut-off angle is close to 90° and does not vary when we change the voltage of the phase winding. Under these conditions, the rectified voltage is proportional to the line voltage and we can assume that α equals s.

3. CHANGE OF RECTIFIED VOLTAGE UPON CHANGE OF LOAD CURRENT

If the line voltage remains unchanged while the load resistance R increases, the load current will decrease at the same time as the voltage on the load should remain constant. Under these conditions, as follows from fig. 4, the rectified voltage E will augment, whereas the cut-off angle will decrease. We must take into account that owing to the decrease of current across the phase winding, the voltage on it will increase somewhat. We will characterize this voltage rise of $U_{\phi m}$ by the coefficient p, which exceeds unity by several percents. The magnitude of p is the greater, the lower the power of the rectifier's transformer and the greater the decrease of the load current:

$$p = U_{\phi m 2}/U_{\phi m 1}. \qquad (6)$$

In the case of cut-off angles that are smaller than 90°, we can with sufficient accuracy consider that the rectified current is proportional to the amplitude of the current impulse across kenotron i_{am} and to the cut-off angle θ (2).
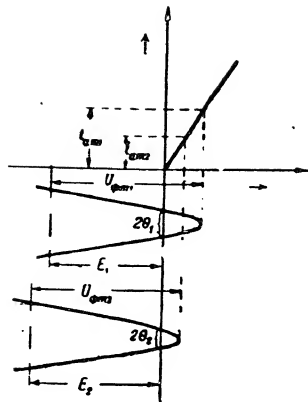
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Fig.4 (Current across kenotron ↑ . Voltage on kenotron →.)

Let us examine two systems occurring upon change of the load resistance. The respective values of the currents and voltages are designated by subscripts 1 and 2.

$$I_{01} = \kappa\, i_{am1}\, \Theta_1, \ I_{02} = \kappa\, i_{am2}\, \Theta_2.$$

Here k is some constant. From figure 5 it follows that:

$$I_{01} = \kappa\, \Theta_1 (U_{\phi m1} - E_1)\, S,$$
$$I_{02} = \kappa\, \Theta_2 (U_{\phi m2} - E_2)\, S,$$

where S is the grid-plate characteristic of the kenotron.

We will characterize increase of the load resistance by the coefficient q, which equals:

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$$q = I_{02}/I_{01}. \tag{7}$$

Under a complete break of load, where the current I_{02} equals zero, $q = 0$. Evidently:

$$q = \frac{\Theta_2}{\Theta_1} \frac{U_{\phi m2} - E_2}{U_{\phi m1} - E_1} = \frac{\Theta_2}{\Theta_2} \frac{p - \beta E_1/U_{\phi m1}}{1 - E_1/U_{\phi m1}}.$$

Here

$$\beta = E_2/E_1 \tag{8}$$

is the coefficient characterizing the augmentation of rectified voltage under break of load. Since E_1 equals $U_{\phi m\,1} \cos \theta_1$, then:

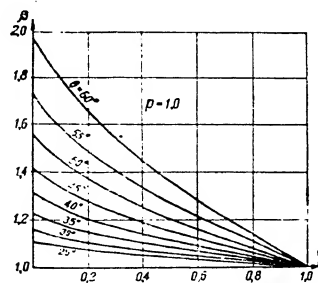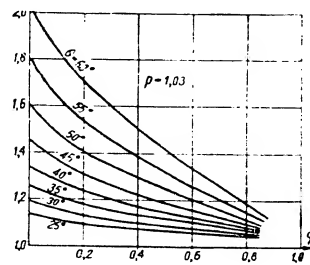$$q = \frac{\Theta_2}{\Theta_1} \frac{1 - \cos \Theta_1}{p - \beta \cos \Theta_1}.$$

Fig.5   Fig.6.

Expressing from this the unknown cutoff angle θ_2, we obtain:

$$\Theta_2 = q \Theta_1 \frac{p - \beta \cos \Theta_1}{1 - \cos \Theta_1}. \tag{9}$$

On the other hand, coefficient β equals:

$$\beta = \frac{E_2}{E_1} = \frac{p U_{\phi m1} \cos \Theta_2}{U_{\phi m1} \cos \Theta_1} = p \frac{\cos \Theta_2}{\cos \Theta_1}.$$

Substituting herein θ_2 from formula 9, we finally get:

$$\beta = \frac{p}{\cos \Theta_1} \cos \left(q \Theta_1 \frac{1 - \cos \Theta_1}{p - \cos \Theta_1} \right). \tag{10}$$

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The sought-for coefficient β is the function of q, for which p and θ_1 are the parameters. From this formula we drew up the graphs shown in figures 5, 6 and 7. From the graphs we see that voltage increase under break of load is especially intensive where q < 0.25 and is larger, the larger the cutoff angle in the kenotron rectifier.

Fig.7.
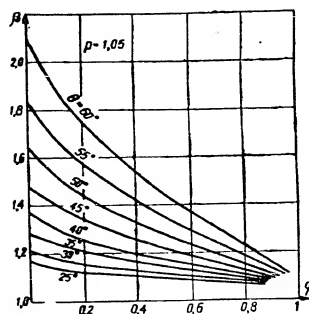
In developing formula 10, the presence of a governing tube in the device had no influence whatsoever. The graphs for coefficient β are also suitable for calculating unstabilized kenotron rectifiers.

4. ORDER OF CALCULATION

The proposed order of calculation is suitable for finding the stabilization limits of the diagram shown in fig.1. To raise the stabilization coefficient, in some circuits, one feeds into the amplifier's input a part of the unstabilized voltage E. The calculatin order does not change here since it refers to the rectifier, the regulating tube and the load.

For the calculation we must pre-assign:

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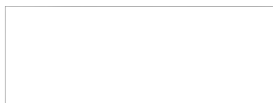
a) The nominal line voltage U, the minimum line voltage U_1, the maximum line voltage U_2;

b) the maximum and minimum values of the stabilized voltage E_{Omax} and E_{Omin}. If adjustment of the stabilized voltage is not required, then E_{Omin} equals E_0;

c) the maximum and minimum values of the load current I_{Omax} and I_{Omin}. If the load does not vary, then I_{Omin} equals I_{Omax}.

The calculation should be made in the following way:

1. We determine the coefficients:

$$s_1 = U_1/U, \qquad (11)$$

$$s_2 = U_2/U, \qquad (12)$$

$$q = \frac{I_{Omin} + \Delta I_e}{I_e + \Delta I_0}, \qquad (13)$$

where ΔI_0 is the current branching off from cathode L_3 and not going into the load. (For example, the current for feeding the volt stabilizer (stabilovolt) in the circuit of the stabilized rectifier itself.)

2. We select, proceeding from a current $I_0 \neq \Delta I_0$, the type and orientative number of regulating tubes. In terms of the characteristics of this tube we determine the minimum voltage drop in tube e_{amin}, proceeding from the current across the tube (the current $I_0 \neq \Delta I_0$ divided by the number of regulating tubes) and the minimum negative displacement on the tube's grid e_c equals $(\theta - 5)$V.

3. We calculate the phase voltage U_{fm1} and the cutoff angle θ_1 in the keno-tron rectifier giving off a current $I_0 \neq \Delta I_0$ and a voltage E_1:

$$E_1 = E_{0\,max} + e_{a\,min} + (I_0 + \Delta I_0) r_L, \qquad (14)$$

where r_L is the effective resistance of the filter's choke (fig.1). We consider the line voltage as equal to U_1.

4. In terms of the calculated cutoff angle θ_1 and in terms of the pre-assigned coefficient s_1, we determine from the graph in fig.2 the coefficient α_1, and then the rectified voltage E occurring at the nominal line voltage equals to U:

$$E = E_1/\alpha_1. \qquad (15)$$

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Under these conditions we will have a cutoff angle θ that is determined from correlation 5. Let us rewrite this expression in the following form:

$$\cos\theta = \frac{a_1}{S_1}\cos\theta_1.\qquad(16)$$

Knowing θ and s_2, we determine the coefficient α_2 from the graph of fig.3. Hence, the rectified voltage E_2 and the cutoff angle θ_2 at the maximum line voltage U_2 will be:

$$E_1 = \alpha_2 E,\qquad(17)$$

$$\cos\theta_2 = \frac{a_2}{s_2}\cos\theta.\qquad(18)$$

5. We assign, in relation to the power of the transformer and the magnitude of coefficient q, an auxiliary coefficient p lying within the limits p equals 1.01 to 1.1. In terms of a known q, θ and p we determine coefficient β (figures 5,6 and 7) and then the maximum rectified voltage E_{max} occurring at the maximum line voltage U_2 and the minimum load current I_{0min}:

$$E_{max} = \beta E_2.\qquad(19)$$

6. The maximum voltage drop on the regulating tubes e_{amax} will occur when, in the conditions of paragraph 5, we establish the minimum stabilised voltage:

$$e_{s\,max} = E_{max} - E_{0\,min} - r_L\,(I_{o\,min} + \Delta I_o).\qquad(20)$$

In terms of the characteristics of the regulating tube we determine the corresponding maximum negative displacement on the grid of lamps e_{cmax}. We must verify whether the obtainment of such a potential is guaranteed in the stabilised rectifier's circuit. In particular, for the widely applied schematic shown in fig.8, the cathode potential of tube $L1$ should be lower than:

$$E_x = E_{o\,min} - e \quad - |e_{c\,max}|.\qquad(21)$$

Here e_{acou} is the anode-cathode voltage of L_4 where the voltage between the grid and cathode equals zero. It is usually small — around 5 to 20 V.

To obtain a high stability, the cathode potential of L_4 should be raised. If this is impossible, it is advisable to hook up an additional stabilovolt, as

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shown in fig.9.

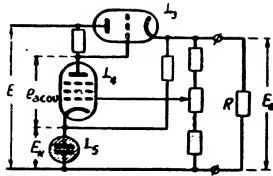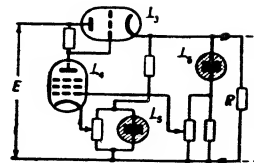Fig.8   Fig.9

7. We make a check on the allowability of dispersion on the anodes of regulating tubes at the power occurring under maximum line voltage U_2, maximum current $I_0 + \Delta I_0$ and minimum stabilised voltage E_{0min}:

$$P_{a\,max} = [E_a - E_{0min} - r_L(I_{0max} + \Delta I_0)](I_{0max} + \Delta I_0). \quad (22)$$

If the power per lamp exceeds the permissible power, we must increase the number of regulating tubes or use more powerful tubes, or, finally, reconsider the technical problem.

8. In calculating, account must be taken of the following:

The rectifier's transformer must, in its construction, be adapted to work at the maximum line voltage U_2, the rectified voltage E_2 and the current I_{0max} ΔI_0. The maximum reverse voltage in the rectifier occurs at line voltage U_2.

The anode-grid space of the regulating tube should be able to bear a voltage of $|e_{a\,max}| + |e_{c\,max}|$.

The condensers of the rectifier's filter should be prepared for a working voltage of E_{max}.

5. PECULIARITIES OF CALCULATING A STABILIZED RECTIFIER IN THE PRESENCE OF A
   SHUNTED REGULATING TUBE.

As was shown in Bibl. 1, the hooking up of a shunt results in a decrease of

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the stabilisation factor and an augmentation of the internal resistance of the stabilized rectifier. This drawback can be eliminated by increasing the amplification factor of the amplifier.

However, the basic drawback of this system is the narrowed stabilisation limits brought into effect by hooking up the shunt.

At the minimum rectified voltage E equals E_{min}, the negative voltage on the grid of the governing tube is minimum, the current I_a passing across the tube is maximum (I_a equals $I_{a\ max}$), whereas the voltage drop e_a on the tube is minimum. In this case we can write out:

$$E_{min} = E_0 + e_{a\ min},$$

$$I_0 = \frac{e_{a\ min}}{R_{sh}} + I_{a\ max}.$$

Hence:

$$R_{sh} = \frac{e_{a\ min}}{I_0 - I_{a\ max}}.$$

Let us introduce coefficient t, which shows what part of the rectified current is passed across the regulating tube (t < 1)

$$t = I_{a\ max} / I_0. \tag{23}$$

Thus

$$R_{sh} = \frac{e_{a\ min}}{I_0(1-t)}. \tag{24}$$

At the maximum voltage E equals E_{max}, the negative voltage on the grid of the governing tube is maximum, the current I_a is minimum (e.g. equal to zero), whereas the voltage drop on the tube e_a is maximum. In this case we can write:

$$E_{max} = E_0 + e_{a\ max}; I_0 = \frac{e_{a\ max}}{R_{sh}}; I_a = 0.$$

Hence:

$$e_{a\ max} = \frac{e_{a\ min}}{1 - t}.$$

Let us designate that:

$$\gamma = E_{max} / E_{min}. \tag{25}$$

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Coefficient γ characterized the width of the stabilization limits of the system. Evidently, the following inequality should be observed:

$$\alpha\beta < \gamma, \tag{26}$$

From the above expression we get:

$$\gamma = \frac{1 + \frac{e_{a\,min}}{E_0(1-t)}}{1 + \frac{e_{a\,min}}{E_0}}, \tag{27}$$

whence:

$$t = \frac{\gamma - 1}{\gamma - \frac{1}{1 + e_{a\,min}/E_0}}. \tag{28}$$

The wider the required stabilization limits (the greater γ is), the greater must be coefficient t, i.e. the greater the amount of rectified current that must be passed through the governing tube.

As shown in Bibl. 1, the maximum dispersion power on the anode of the governing tube occurs at a rectified voltage equal to:

$$E_{cr} = E_0 + \frac{R_{w}\,I_0}{2}.$$

Substituting herein R_{sh} from formula 24, we get:

$$E_{\kappa p} = E_0 + \frac{e_{a\,min}}{2(1-t)}. \tag{29}$$

E_{cr} is always less than E_{max}. If t > 0.5, then $E_{cr} > E_{min}$ and the dispersion power on the anode reaches a maximum at E = E_{cr}:

$$P_{a\,max} = \frac{I_0^2\,R_{w}}{4} = \frac{I_0\,e_{a\,min}}{4(1-t)}. \tag{30}$$

If t < 0.5, then $E_{cr} < E_{min}$. Under these conditions, the dispersion power is greatest at the maximum voltage E equals E_{min}:

$$P_{a\,max} = t I_0\,e_{a\,min} \tag{31}$$

Calculation of the stabilization limits of a stabilized rectifier with a shunt is conducted basically the same as without a shunt (section 4) except for the following differences.

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a) in point 2, the type, number of regulating tubes and voltage drop in them $e_{a\ min}$ are not determined in terms of the total current $I_0 + \Delta I_0$ but in terms of the anode current of the governing tube:

$$I_{a\ min} = t(I_0 + \Delta I_0).$$

The magnitude of t should be determined from formula 28 in terms of the orientative magnitude of γ, close to the product of $\alpha_1 \alpha_2 \beta$.

b) After determining, in points 2, 3, 4, and 5, the coefficients $\alpha_1 \alpha_2$ and β, it must be made certain that the following correlation is fulfilled:

$$\alpha_1 \alpha_2 \beta < \gamma.$$

If this is not so, one must alter coefficient t.

Here we must calculate γ from a formula that is more accurate than formula 27:

$$\gamma = \frac{1 + \dfrac{e_{a\ min}}{E_0} \dfrac{1 - t\,\dfrac{I_{a\ min}}{I_{a\ max}}}{1 - t}}{1 + \dfrac{e._{min}}{E_0}}, \tag{30}$$

where

$$I_{a\ min} = (0,1 \div 0,2) I_{a\ max}.$$

c) When determining, in point 6, the voltage drop on the tube $e_{a\ max}$ and the voltage on the grid $e_{g\ max}$, we must substitute the current $I_{a\ min}$ instead of the current $(I_0 + \Delta I_0)$.

d) The check on the dispersion power on the anodes of the regulating tubes is done under the conditions of point 7. Here the power is calculated from formula 30 of 31, depending on the magnitude of t.

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NARROW-BAND QUARTZ FILTERS FOR INTER-TUBE CONNECTIONS

(The article presents a method for calculating narrow-band quartz filters for inter-tube connections residing in the duplication of the schematic characteristics of phase circuits. The filters are constructed in terms of an asymmetrical system using quartz resonators with divided electrodes.

The proposed method makes it possible to determine what characteristics of effective attenuation and phase reversal can be obtained in narrow-band quartz filters, which considerably facilitates their calculation and the selection of optimum parameters.

Formulas are given for calculating the characteristics of narrow-band quartz filters, taking into account losses in the resonators.)

In the technology of communication it is often necessary to separate an extremely narrow frequency spectrum or a single frequency with the aim of analyzing or adjusting the signal. It is for this purpose that we use narrow-band quartz filters.

Narrow-band filters can be constructed in terms of a symmetrical or an asymmetrical scheme. For inter-tube connections, it is preferable to construct narrow band filters in the asymmetrical scheme since in this case there is no need to assure symmetrical loads for the filter.

The availability of industrially designed, highly stable vacuum quartz resonators with divided electrodes (fig.1a) facilitates the construction of such filters.

An equivalent diagram of the quartz resonator is given in fig.1b. It possesses two resonance frequencies:

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$$f_1 = \frac{1}{2\pi\sqrt{L_1 C_1}}; \quad f_{ant} = f_1 \sqrt{1 + \frac{C_1}{C_2}}.$$

Fig.1
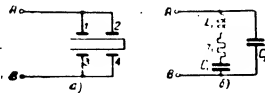
The magnitude of the ratio C_1/C_2 is determined by the piezoelectric properties of the quartz and satisfies the condition:

$$\frac{C_1}{C_2} < 0.0072;$$

therefore, with a high degree of accuracy:

$$\frac{\Delta f_{res}}{f_1} = \frac{C_1}{2C_2} < 0.0036,$$

where

$$\Delta f_{res} = f_{ant} - f_1.$$

Joining to terminals A and B the capacity ΔC_2, we can unlimitedly approach the anti-resonance frequency to the resonance frequency without varying f_1 in this process. Let the capacity joined parallel to C_2 be equal to ΔC_2 and the new anti-resonance frequency equal to f_2. Then:

$$\Delta f_{12} = \Delta f_{res} \frac{C_2}{C_2 + \Delta C_2}, \tag{1}$$

where $\Delta f_{12} = f_2 - f_1$.

The capacity C_2 is composed of the static capacity between the metallicized layers 1—3 and 2—4 of the quartz plate and the capacity between the lead-out wires of the quartz-holder. In constructing resonators one tends to make the latter capacity as small as possible so as to obtain a maximum difference of Δf_{12}.

Fig.2. - see next page

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Fig.2.
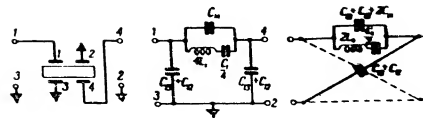
The equivalent diagram of the resonator contains also a loss resistance, r_1; however, its magnitude is such that its rating is calculated in tens of thousands of units and, at the least, satisfies the condition:

$$Q_1 = \frac{\omega_1 l_1}{r_1} > 15\,000.$$

Hereafter we will not present the resistance r_1, assuming that the resonator's equivalent diagram represents an ideal three-element circuit. The influence of losses on the filter's performance will be taken into account separately.

The equivalent diagram of the quartz resonator with divided electrodes under conditions of asymmetric connection (Bibl.3) is given in fig.2.

The capacity $C_{13} = C_{24} = \frac{C_2}{2}$.

The capacities C_{12} and C_{14} are composed of the static capacity between the corresponding metallisation layers and the capacity between the lead-out wires of the quartz-holder. Here we pre-suppose that the resonator is constructed in such a way that capacity $C_{12} = C_{34}$.

The static capacity between metallisation layers 1—2 or 1—4 is inappreciable. The capacity between lead-out wires 1—4 is smaller than the capacity between lead-out wires 1—2, which is caused by the construction of the quartz-holder, in which the distance between lead-out wires 1—4 is greater than between 1—2.

The presence of parasite capacities 1—2 and 1—4 lowers the anti-resonance frequency of the series arm of the bridge diagram in fig.2 in comparison with the diagram in fig.1. The magnitude of the difference will be:

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$$\Delta f_{12} = \Delta f_{res} \; \frac{C_{13}}{C_{13} + C_{19} + 2\,C_{14}}. \tag{2}$$

Augmenting the capacities of C_{13} and C_{24} by hooking up additional condensers $\Delta C_{13} = \Delta C_{24}$, we can vary Δf_{12} to an unlimitedly small magnitude; here:

$$\Delta f_{12} = \Delta f_{res} \; \frac{C_{13}}{C_{13} + C_{19} + 2\,C_{14} + \Delta C_{13}} \tag{3}$$

We know that the diagram in fig.2 represents a band filter with maximum frequencies of f_1 and f_2.

Thus, by augmenting the magnitudes of C_{13} and C_{24} by hooking up $\Delta C_{13} = \Delta C_{24}$ on the side of the input and output, it is easy to obtain the necessary difference of maximum frequencies, in this way adjusting the effective band of filter passage in conformity with the pre-assigned requirements.

The simplest formulas for calculating the characteristics and elements of a filter are obtained when using the method of forming band filters by means of asymmetrical diagram transformation of phase circuits of the μ type (1).

<u>Fig.3.</u>
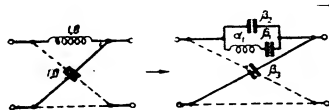
As we know, the bridge diagram in fig.2 can be obtained as the result of the asymmetrical transformation of a μ-type phase circuit. If we adopt that $\mu = 1$, i.e. if we study the transformation of an elementary phase circuit having normalised inductance and capacity constants equal to unity, we obtain a bridge diagram (fig.3) in which the conductance of the series arm:

$$Y_a = \frac{1}{tr}\, p' \; \frac{p'^2 + x_2^2}{p'^2 + x_1^2} = \frac{1}{\tau r}\, p' + \cfrac{1}{\cfrac{\tau r}{x_2^2 - x_1^2}\, p' + \cfrac{1}{\cfrac{x_2^2 - x_1^2}{\tau r\, x_1^2}\, p'}}.$$

while the conductance of the parallel arm:

$$Y_b = \frac{1}{r}\, p'.$$

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Evidently:

$$\beta_2 = \frac{1}{\tau r} \; ; \; \beta_3 = \frac{\tau}{r} \; ; \; \beta_1 = \frac{x_2^2 - x_1^2}{\tau r \, x_1^2} \; ; \tag{4}$$

$$\alpha_1 = \frac{\tau r}{x_2^2 - x_1^2} \, ,$$

whence:

$$\tau = \sqrt{\frac{\beta_3}{\beta_2}} \; ; \; \frac{1}{r} = \sqrt{\beta_2 \beta_3} \; ; \tag{5}$$

$$x_1 = \frac{1}{\sqrt{\alpha_1 \beta_1}} \; ; x_2 = x_1 \sqrt{1 + \frac{\beta_1}{\beta_2}} \, .$$

From a comparison of figures 2 and 3, we see that their characteristics are identical if we assume that:

$$C_{12} + C_{13} + 2\,C_{14} = \beta_2 \, C; \; 2\,L_1 = \alpha_1 \, L; \tag{6}$$

$$C_{12} + C_{13} = \beta_3 \, C; \; \frac{C_1}{2} = \beta_1 \, C,$$

where:

$$L = \frac{R}{\omega_0} \; ; \; C = \frac{1}{R\,\omega_0} \; ;$$

$$\omega_0 = 2\,\pi\,f_0; \; f_0 = \sqrt{f_1 f_2} \; ; \; x_2 = \frac{f_2}{f_0} \; ; \; x_1 = \frac{f_1}{f_0} \, .$$

The filter's frequencies are bound up with the frequencies of the phase circuit by the correlation:

$$p = \tau \sqrt{\frac{p'^2 - x_1^2}{p'^2 + x_2^2}} \, . \tag{7}$$

At the two corresponding frequencies, p and p', the transmission constants of the filter and the phase circuit are identical; we therefore say that the characteristic of the filter's transmission constant duplicates the characteristic of the phase circuit's transmission constant from which it is formed.

The characteristic of the filter's natural damping in the retention band is obtained owing to the characteristic duplication of the phase circuit's natural damping for real values of p(p equals d). Since p' equals ix, then:

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$$d = \gamma \sqrt{\frac{x^2 - x_1^2}{x^2 - x_2^2}} \;. \tag{8}$$

The frequencies of the filter's retention band situated lower than the pass-through band $(0 < x < x_1)$, correspond to the frequencies of the phase circuit on segment $\gamma x_1^2 \div 0$. This follows directly from formula 7, where it must be assumed that p' varies from 0 to ix_1.

It can be shown analogically that the frequencies of the filter's retention band situated above the pass-through band $(x_2 < x < \infty)$, corresponds to the frequencies of the phase circuit on segment $\infty \div \gamma$ of a real axis.

The characteristic of the filter's phase constant in the pass-through band is obtained owing to the duplication of the circuit phase constant's characteristic occurring at imaginary values of p, i.e. real frequencies y.

Since p' equals ix, then:

$$y = \gamma \sqrt{\frac{x^2 - x_1^2}{x_2^2 - x^2}} \;. \tag{9}$$

The damping of the elementary phase circuit at frequency d is equal to:

$$b_s = \ln \left| \frac{1 + d}{1 - d} \right|, \tag{10}$$

whereas the phase constant of the elementary phase circuit at frequency y is equal to:

$$a_s = 2 \operatorname{acrtg} y. \tag{11}$$

Since the frequency difference $f_2 - f_1$ does not exceed 0.0036 f_1, the magnitudes of x_1 and x_2 are very close to unity. In this connection, the above formulas can be considerably simplified. Let us designate:

$$f - f_0 = \delta f; \quad \frac{f_2 - f_1}{2} = \Delta f; \quad \eta = \frac{\delta f}{\Delta f} \;.$$

We will assume that $x_2 + x \approx x_1 + x; \quad f_0 \approx \frac{f_1 + f_2}{2};$

$$\gamma = \sqrt{\frac{\beta_2}{\beta_3}} = \sqrt{\frac{C_{12} + C_{13}}{C_{12} + C_{13} + 2C_{14}}} \approx 1 - \frac{C_{14}}{C_{13} + C_{12} + 2C_{14}} \approx 1, \tag{12}$$

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since

$$C_{14} \ll C_{12} + C_{13}.$$

Incorporating the approximations, we obtain:

$$d = \tau \sqrt{\frac{\eta + 1}{\eta - 1}}, \qquad (13)$$

or

$$y = \sqrt{\frac{1 + \eta}{1 - \eta}}, \qquad (14)$$

$$\eta = \frac{d^2 + \tau^2}{d^2 - \tau^2} \qquad (15)$$

for the retention band, and:

$$\eta = \frac{y^2 - 1}{y^2 + 1} \qquad (16)$$

for the pass-through band.

The magnitude of Δf represents the distance from the middle of the pass-through band f_0 to the maximum frequency, whereas the magnitude of δf is the distance from the middle of the pass-through band to the investigated frequency. Under these conditions, at the middle frequency of the band filter $\eta = 0$, while at the maximum frequencies $\eta = \pm 1$. Thus, the frequencies of the quartz filter are bound up with the frequencies of the phase circuit by a very simple relationship.

Fig.4.
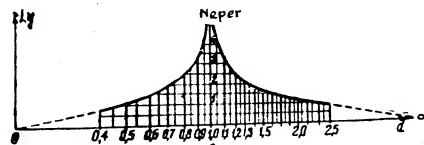
Assigning ourselves the values of d and y, from formulas 10 and 11 it is easy to determine the damping and phase constant, while from formulas 15 and 16 — the corresponding frequencies of the filter in scale η. In fig.4 we show the damping characteristic of the elementary phase circuit on real axis d of plane p. On axis d is plotted the logarithmic scale in relation to the point d equals 1.

In view of the damping symmetry of the elementary phase circuit with respect

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to point d equals 1, at two frequencies of the filter corresponding to mutually converse values of d and situated on both sides of the pass-through band, the damping will be identical.

The characteristic resistance of the filter equals:

$$Z = \frac{r}{p'} \cdot \sqrt{\frac{p'^2 + x_1^2}{p'^2 + x_2^2}} .$$

In order for the characteristic resistance at the middle frequency of the pass-through band to be equal to the load resistance, it is necessary, as we know to assume r equal to x_2. Under these conditions, the load resistance is adopted as equal to unity (1). As a result:

$$Z = -i \frac{x_2}{x} \cdot \sqrt{\frac{x^2 - x_1^2}{x^2 - x_2^2}} . \quad (17)$$

Making use of formulas 8 and 9, and also the approximations introduced above, it is easy to show that for the pass-through band, with a great degree of accuracy:

$$Z = y. \quad (18)$$

Analogically, for the characteristic resistance in the retention band, we get:

$$Z = -i \frac{d}{x} . \quad (19)$$

In the vicinity of the pass-through band, with a great degree of accuracy:

$$Z = -id . \quad (20)$$

When we connect a filter into the circuit, and particularly into the inter-stage connection, its characteristics will be determined by the working parameters determined by the characteristic parameters and load resistances.

In the pass-through band, the effective damping is also influenced by losses in the filter's elements. Hereafter, however, we will designate by b_p the effective (operating) damping without regard taken for losses. The effective damping with regard taken for losses will be designated by $b_p + b_{loss}$.

To determine the effective damping and effective phase reversal of the symmetrical diagram (1) it is convenient to make use of the following expression:

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$$e^{2b_p} = 1 - \frac{1}{4}\left(Z - \frac{1}{Z}\right)^2 sh^2 g = ch^2 g - \frac{1}{4}\left(Z + \frac{1}{Z}\right)^2 sh^2 g. \tag{21}$$

This expression can evidently be presented in the form of the product of two conjugated multipliers:

$$e^{2b_p} = e^{g_p} \cdot e^{\overline{g}_p},$$

where

$$e^{g_p} = ch\, g + \frac{1}{2}\left(Z + \frac{1}{Z}\right) sh\, g, \tag{22}$$

$g_p = b_p + ia_p$ is the working measure of transmission,

$g = b + ia$ is the transmission constant.

Hereafter we will examine a system consisting of identical elementary links obtained, for example, through the cascade connection of resonators (fig.9). The transmission constant of such a system in the retentive band equals $Nb_\mathfrak{s} + iN\pi$ up to the pole of natural damping and $Nb_\mathfrak{s} + i0$ at frequencies situated above the pole of natural damping, where $b_\mathfrak{s}$ is the natural damping of one elementary link. Therefore, in the retention band chg equals chb, shg equals shb we can write:

$$e^{g_p} = ch\, Nb_\mathfrak{s} + \frac{1}{2}\left(Z + \frac{1}{Z}\right) sh\, Nb_\mathfrak{s}.$$

Since for dampings occurring in the guaranteed retention band:

$$ch\, Nb_\mathfrak{s} \approx sh\, Nb_\mathfrak{s} \approx \frac{e^{Nb_\mathfrak{s}}}{2},$$

then:

$$e^{g_p} = e^{Nb_\mathfrak{s}}\left(\frac{Z+1}{2\sqrt{Z}}\right)^2 = e^{Nb_\mathfrak{s} + 2b_{neq} + ia_p}, \tag{23}$$

where:

$$b_{neq} = \ln\left|\frac{Z+1}{2\sqrt{Z}}\right|; \quad a_p = 2\arc\frac{Z+1}{2\sqrt{Z}}.$$

Therefore:

$$b_p = Nb_\mathfrak{s} + 2b_{neq}.$$

According to formula 19, in the retention band, Z equals $-i\frac{d}{x}$, while in the guaranteed retention band situated in the vicinity of the pass-through band of the narrow-band filter, $x \approx 1$, whereupon d approaches unity; we therefore get:

$$\cdot\, b = \ln\left|\frac{1+i1}{2\sqrt{i1}}\right| = -0,345\; \text{nep}.$$

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Thus, we have a highly simple expression for the effective damping of the
narrowband filter in a large frequency range (large in terms of absolute magni-
tude and also the most interesting range for us) of the retention band, namely:

$$b_p = Nb_s - 0.69 \text{ nep} \qquad (24)$$

Outside this region, as approach is made toward x = 0 and x = ∞:

$$b_{o=p} = \ln \left| \frac{1 - i\frac{d}{x}}{2\sqrt{-i\frac{d}{x}}} \right|, \qquad (25)$$

and therefore the damping effect conditioned on reflection can only increase,
since d → 1. Where x equals 0 and x equals ∞ we have an effective damping pole
of the filter that is independent of the natural damping pole conditioned on the anti-
resonance frequency of the resonator (fig.2).

Let us determine the effective damping of the narrow-band filter in the
pass-through band. Since, in the pass-through band, g equals 0 plus iNa, i.e.
b equals 0, then:

$$e^{g_p} = \cos Na_s + i\,\frac{1}{2}\left(Z + \frac{1}{Z} \right) \sin Na_s,$$

$$a_p = 1 + \frac{1}{2}\left(Z - \frac{1}{Z} \right)^2 \sin^2 Na_s, \qquad (27)$$

$$a_p = \operatorname{arctg}\left[\frac{1}{2}\left(Z + \frac{1}{Z} \right) \operatorname{tg} Na_s \right]. \qquad (28)$$

According to formulas 18 and 11, Z equals y equals tg $\frac{a_p}{2}$, and therefore we
can write:

$$e^{2b_p} = 1 + \frac{\sin Na_s}{\operatorname{tg}^2 a_p}, \qquad (29)$$

$$a_p = \operatorname{arctg}\frac{\operatorname{tg} Na_p}{\sin a_p}. \qquad (30)$$

Since the limit frequencies of the filter x_1 and x_2 correspond to frequencies
y equals 0 and y equals ∞ in the phase circuit, at limit frequencies a, equals 0
and a, equals π; therefore, as approach is made toward the limit frequencies,
expression 29 approaches the limit:

$$e^{2b_p} = 1 + N^2.$$

whereas expression 30 approaches the limit a_p equals arctgN for x equals x_1

and a_p equals $N\pi -$ arctg$N$ for x equals x_2.

Consequently, at the limit frequenc es the effective damping is determined from the formula:

$$b_p = \frac{1}{2}\ln(1 + N^2).$$

From formula 29 we see that at those frequencies of the pass-through band at which $\sin Na_2$ equals 0, i.e. a_2 equals $\frac{\nu\pi}{N}$, where ν equals 1, 2, 3...N - 1, the effective damping equals zero.

Since y equals tg $\frac{a_2}{2}$, then:

$$\eta = \frac{y^2 - 1}{y^2 + 1} = -\cos a_2, \tag{31}$$

and therefore the effective damping equals zero where:

$$\eta = -\cos\frac{\nu\pi}{N}. \tag{32}$$

If we assume that $\quad \sin Na_2 = \pm 1$, then a_2 equals $\frac{(2\nu - 1)}{2N}$, where ν equals 1, 2...N. Under these conditions:

$$b_p = \ln\frac{1}{\sin\frac{2\nu - 1}{2N}\pi} \tag{33}$$

This damping magnitude occurs where

$$\eta = -\cos\frac{(2\nu - 1)}{2N}\pi. \tag{34}$$

Making use of the resultant formulas, we will write out expressions for b_p where N equals 1, 2, 3.

For N = 1:

$$b_p = \frac{1}{2}\ln(1 + \eta^2). \tag{35}$$

For N = 2:

$$b_p = \frac{1}{2}\ln(1 + 4\eta^4). \tag{36}$$
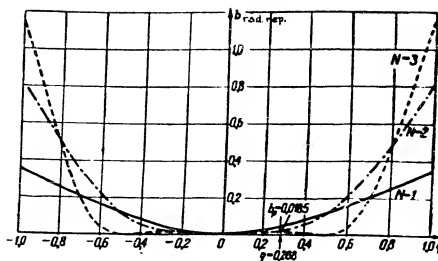
For N = 3:

$$b_p = \frac{1}{2}\ln[1 + \eta^2(4\eta^2 - 1)^2]. \tag{37}$$

In fig.5 we give the characteristics of effective damping in the pass-through

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band for N equals 1, 2, 3. On the x-axis is plotted the η scale.

Fig.5.

Regardless of the number of elementary links, for the given value of $|\eta| < 1$, the maximum damping magnitude is determined from the following correlation:

i.e.
$$e^{2b_p} = 1 + \frac{1}{tg^2 a_o} = \frac{1}{\sin^2 a_o} = \frac{1}{1-\eta^2},$$

$$b_p = \frac{1}{2} \ln \frac{1}{1-\eta^2}.$$

Therefore, in the most unfavorable case, where $\sin Na_3$ equals ± 1 and where $\eta \pm 0.71$, irregularity of damping in the frequency band $-0.71 < \eta < 0.71$ does not exceed 0.35 nep. The frequency band with such damping irregularity is the effective band for many practical problems.

In the narrow-band filter's equivalent diagram, we assumed that there are no energy losses in the resonator. In actuality, as we see from fig.1, in series with the resonance circuit $L_1 C_1$, there is hooked up a loss resistance that can be referred over to the inductance L_1, and we can assume that the capacities of the resonator's equivalent diagram are ideal, whereas the inductance has a figure of

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merit $Q_1 = \frac{\omega_1 L_1}{r_1}$.

Where there are losses in the filter's elements, the working measure of transmission can be presented in the form of a converging series:

$$g_p'(p', d) = g_p(p') + \frac{\partial g_p(p')}{\partial p'} + \frac{1}{2} \frac{\partial^2 g_p(p')}{\partial p'^2} d^2 + \cdots ,$$

where (1):

$$d = \frac{d_1 + d_0}{2} = \frac{1}{2} \left(\frac{1}{Q_1} + \frac{1}{Q_2} \right) = \frac{1}{2Q} , \qquad (38)$$

$g_p(p', d)$ is the working measure of transmission in the presence of losses,

$g_p(p')$ is the working measure of transmission in the absence of losses,

Q_1 is the figure of merit of inductances,

Q_2 is the figure of merit of capacitances.

In our case, $1/Q_2 \approx 0$, and therefore Q equals Q_1.

Since particular points of the transmission measure are only the damping poles, the convergence of this series occurs up to frequencies situated in the vicinity of d of the damping pole. Thus, the convergence region of the function series of the transmission measure is considerably wider than that of the function series of the transmission constant. Where the damping pole is sufficiently distance from the maximum frequency of the filter, the resultant series also converges at the maximum frequency. Limiting ourselves to two terms from series 38, we get:

$$g_p(p', d) = b_p + d \frac{\partial a_p}{\partial x} + i \left(a_p - d \frac{\partial b_p}{\partial x} \right).$$

Thus, owing to losses in the elements, the effective damping in the pass-through band increases by the following magnitude:

$$b_{noise} = d \frac{\partial a_p}{\partial x} = \frac{1}{2Q} \cdot \frac{\partial a_p}{\partial x}. \qquad (39)$$

When the operating phase reversal in the pass-through band is little different from the natural phase reversal, we can write:

$$b_{noise} = \frac{1}{2Q} \cdot \frac{\partial a_p}{\partial a} \cdot \frac{\partial a}{\partial x} \approx \frac{1}{2Q} \cdot \frac{\partial a}{\partial x}. \qquad (40)$$

For band filters with an input resistance class $\geqslant 2$, this formula is adequate since the echo damping in the effective pass-through band is large enough so that

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$\frac{1}{4} \cdot \left(Z + \frac{1}{Z} \right) \approx 1$; therefore, according to formula 28, $a_p \approx a$.

In a narrowband quartz filter, the characteristic resistance class equals 1, and therefore the operating phase reversal is greatly different from the natural phase reversal. The characteristics of the operating and natural phase reversal of a narrow-band quartz filter for N equals 1 is given in fig.6.

Using expression 30, it is easy to determine:

$$\frac{\partial a_p}{\partial a_s} = \frac{2N \sin a_s - \cos a_s \sin 2N a_s}{2 \sin^2 a_s \left(1 + \frac{\sin^2 N a_s}{\lg^2 a_s} \right)} \qquad (41)$$

and further

$$\frac{\partial a_p}{\partial a} = \frac{1}{N} \cdot \frac{\partial a_p}{\partial a_s}, \qquad \text{since a equals } Na_3 .$$

In addition, as we know (1), for the elementary link of a band filter:

$$\frac{\partial a_s}{\partial x} = \frac{x \left(x_2^2 - x_1^2 \right)}{\left(x^2 - x_1^2 \right) \cdot \left(x_2^2 - x^2 \right)} \cdot \frac{2y}{1 + y^2} \qquad (42)$$

and further:

$$\frac{\partial a}{\partial x} = N \frac{\partial a_s}{\partial x} .$$

Since the frequencies of a band filter are bound up with the frequencies of a phase circuit by the correlation (1):

$$x = \sqrt{\frac{\gamma^2 x_1^2 - p^2 x_2^2}{\gamma^2 - p^2}} = \sqrt{x_1^2 + (x_2^2 - x_1^2) \cdot \frac{p^2}{p^2 - \gamma^2}} .$$

then for the pass-through band, assuming that p equals iy and $\gamma \approx 1$, we have:
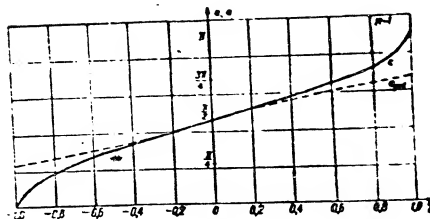
$$x^2 - x_1^2 = \left(x_2^2 - x_1^2 \right) \cdot \frac{y^2}{y^2 + 1} ,$$

$$x_2^2 - x^2 = \left(x_2^2 - x_1^2 \right) \cdot \frac{1}{y^2 + 1}$$

Fig.6.

See next page

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Fig. 6.

Consequently:

$$\frac{\partial a_s}{\partial x} = 2n \frac{1}{\sin a_s},$$
(43)

where n equals $\frac{1}{x_2 - x_1}$ equals $\frac{f_0}{f_2 - f_1}$ — is the ratio of the average frequency of the filter's pass – through band to the difference of maximum frequencies.

Making use of the resultant expressions, we can write:

$$b_{noise} = \frac{n}{Q} \cdot \frac{2N \sin a_s - \cos a_s \sin 2Na_s}{2\sin^2 a_s \left(1 + \frac{\sin^2 Na_s}{\lg^2 a_s}\right)}.$$
(44)

For the average frequency (y equals 1) a_s equals $\frac{\pi}{2}$, and therefore:

$$b_{noise} = N \frac{n}{Q}.$$
(45)

For the maximum frequencies, revealing the indefiniteness in expression 44, we find that:

$$b_{noise} = \frac{n}{Q} \frac{N + 2N^3}{3(1 + N^2)}.$$
(46)
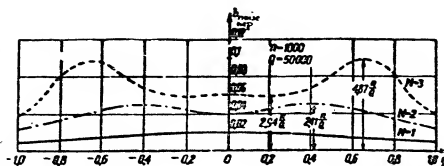
For frequencies at which $\sin Na_s = 0$,

$$b_{noise} = N \frac{n}{Q} \cdot \frac{1}{\sin^2 a_s}.$$
(47)

For frequencies at which $\sin Na_s = \pm 1$,

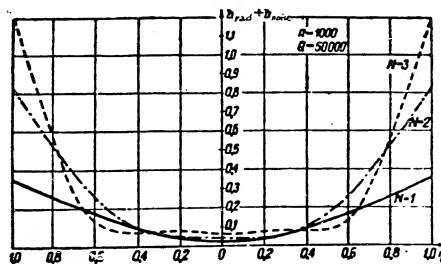86

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$$b_{noise} = N \frac{n}{Q} . \qquad (48)$$

Fig.7.

For N equals 1, 2, 3 we can write out highly simple formulas for calculating the damping effect conditioned on the losses:

$$
\left.
\begin{aligned}
b_{noise} &= \frac{n}{Q} \cdot \frac{1}{1+\eta^2} && \text{for } N=1 \\
b_{noise} &= \frac{n}{Q} \cdot 2 \frac{1+2\eta^2}{1+4\eta^4} && \text{for } N=2 \\
b_{noise} &= \frac{n}{Q} \frac{3+16\eta^4}{1+\eta^2(4\eta^2-1)^2} && \text{for } N=3
\end{aligned}
\right\}
\qquad (49)
$$

Fig.8.

In fig.7 we give the damping characteristics of a narrow-band quartz filter, as conditioned on the losses in the resonators, while in fig. 8 we give the sum

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of dampings of narrow-band quarts filters in the pass-through band, as conditioned on reflection and losses in the resonators.

The afore-established approximations also permit a considerable simplification of the formulas for calculating the elements of fig.3.

Thus, formulas 4 can be rewritten in the following form:

$$\beta_2 \approx 1; \ \beta_3 \approx 1; \ \alpha_1 \approx \frac{n}{2}; \ \beta_1 \approx \frac{2}{nx_1^2} \approx \frac{2}{n}. \quad (50)$$

The cut type and size of the quartz plate for a narrow-band filter are determined, when designing the resonator, by a number of considerations bound up with the temperature coefficient, the absence of additional resonance frequencies close to the basic one, by the production conditions,etc. Therefore, when designing narrow-band quarts filters one can widely vary the magnitude of the equivalent inductance. However, after the final parameters of the resonator have been adopted, they must be exactly repeated in the subsequent production so that the filter connected between the adopted load resistances might have characteristics that are close to the rated ones.

Fig.9
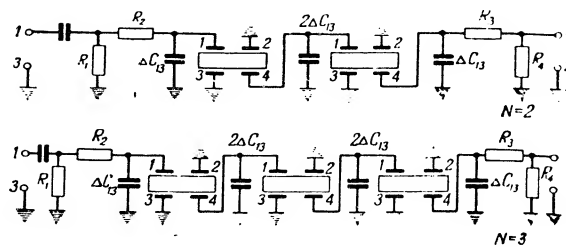
The load resistances are determined in the following way:

Let us take a given resonator. Having measured the capacitances of C_{12}, C_{13} and C_{14}, and also of Δf_{res}, we can determine with formula 3 the additional cap-

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acitance of ΔC_{13}, which must be included in the circuit in order to obtain the necessary difference of limit frequencies Δf_{12}. Further, from formulas 6 and 50 we get:

$$C = \sqrt{(C_{12} + C_{13} + \Delta C_{13} + 2C_{14})(C_{12} + C_{13} + \Delta C_{13})} \approx \qquad (51)$$
$$\approx C_{12} + C_{13} + \Delta C_{13} + C_{14}.$$

Since the unit capacitance C equals $\dfrac{1}{R\,\omega_0}$, the load resistance:

$$R = \frac{1}{\omega_0 C} .$$

The filter's diagram for inter-tube connections, this diagram consisting of two and three cascade-connected resonators, is shown in fig.9. The resistances R_1, R_2, R_3 and R_4 satisfy the condition:

$$R_1 + R_2 = R_3 + R_4 = R,$$

where R_1 is the load resistance of the anode circuit, and R_4 the resistance connected between the grid and ground of the next cascade. The ratio R_1/R and R_4/R can be selected with such a magnitude that the anode-ground and grid-ground tube capacitances, recalculated into the filter's diagram, are less than the capacitance of ΔC_{13}.

The damping by voltage between input 1 and output 4 of the inter-tube connection equals:

$$B = \ln \frac{R_2 + R_3 + R_4}{R_4} + b_{rad} + b_{noise} \qquad (52)$$

Example

Let us say that we must construct a narrow-band quartz filter for inter-tube connections having the following characteristics:

1. Effective pass-through band of the filter 40 cps where the middle frequency is situated within $50,000 \pm 25$ cps. The filter's damping difference at the extreme frequencies of the effective pass-through band and at the middle frequency should not exceed 0.35 nep.

2. It is desirable that the damping irregularity in the frequency band $f_0 \pm$

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$\pm$ 10 cps. should not exceed 0.02 nep.

3. The damping in the retention band at a frequency that is δf equals $\pm$ $\pm$ 280 cps. away from the middle frequency should be $\geqslant$ 7 nep.

4. The filter should be constructed in an asymmetrical scheme. Let us assign ourselves a coefficient of pass-through band use of 0.71. Then:

$$f_2 - f_1 = \frac{40}{0.71} = 56 \; cps$$

To construct the filter we use a resonator with a 5° cut since the temperature coefficient of such a resonator does not exceed 1.5—2.10^{-6}.

Let us say we have resonators with a resonance frequency of 55,975 cps. Q_1 equals 50,000, C_{13} equals C_{24} equals 7 f, C_{12} equals 2.7 f, C_{14} equals 0.8 f, Δf_{res} equals 196 cps., Δf_{12} equals 121 cps.

Since the middle frequency of the filter equals 55,975 + 28 cps. equals 56,003 cps. and is found in the pre-assigned limits of 50,000 $\pm$ 25 cps., the resonator can be adopted for calculating the assigned filter (Footn.a). Δf_{12} = 121 cps. is the maximum difference between the filter's limit frequencies that can be obtained in the given circuit through the use of the given type of resonator. Since this difference exceeds the required one, then by means of adding a supplementary capacitance ΔC_{13} we can decrease it to 56 cps.

Making use of correlation 3, we determine that ΔC_{13} = 13.3 f. With the aid of formula 12 we determine that γ equals 0.9675, which proves to be very close to unity.

The projected circuit should possess an operating damping of 7 nep. where:

$$\eta = \pm \frac{280}{28} = \pm 10,$$

which in the plane of the phase circuits corresponds to:

$$d = \gamma \sqrt{\frac{\eta + 1}{\eta - 1}} = \gamma \, 1.108^{\pm 1}.$$

Since the parameter of γ is close to unity, d has approximately two mutually reverse values. From the graph in Fig.4 we see that for these values of d, the

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damping of the phase circuit equals 3.0 nep. Since to obtain an operating damping
of 7nep. we must, according to formula 24, have a natural damping of 7.69 nep.,
then evidently we must use at least three elementary links or, which is the same
thing, three resonators.

Calculation of the characteristics can be done easiest in the following manner:
we set ourselves an operating damping of the filter and determine the frequencies
at which it occurs.

The filter has three frequencies of infinite natural damping. One frequency
coincides with the frequency of infinite natural damping occurring when the re-
sistances of the bridge's arms are equal. In the plane of the phase circuits it
occurs where d equals 1, and therefore, according to formula 15:

$$\eta_\infty = \frac{d^2 + \gamma^2}{d^4 - \gamma^2} = \frac{1 + \gamma^2}{1 - \gamma^2} = 29.8.$$

Consequently:

$$\ell f_\infty = \Delta f \cdot \eta_\infty = 29.8 \cdot 28 = 835 \text{ cps}$$

Two frequencies of infinite operating damping occur where x equals 0 and x
equals ∞ since at these frequencies the characteristic resistance equals ∞
or 0; therefore, the damping of reflection equals ∞.

Let us determine the frequencies at which the operating damping equals 7 nep.,
i.e. the natural damping equals 7.69 nep.

Since all links are identical, the natural damping of one link equals 2.57 nep.
The phase circuit has a damping of 2.57 nep. at two frequencies situated symme-
trically in relation to the pole.

Since the pole is situated at point d equals 1, these two frequencies satisfy
the condition:

$$d_b \text{ equals } \frac{1}{d_a},$$

where d_b and d_a are frequencies at which there is identical natural damping of the

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phase circuit.

According to formula 10:

$$d_a = \frac{e^b - 1}{e^b + 1} = 1 - \frac{2}{e^b + 1} = 0.857,$$

$$d_b = \frac{1}{d_a} = 1.167.$$

Consequently according to formula 15:

$$\tau_a = \frac{\delta f_a}{\Delta f} = \frac{d_a^2 + \gamma^2}{d_a^2 - \gamma^2} = -8.3$$

and

$$\nu_b = \frac{\delta f}{\Delta f} = \frac{d_b^2 + \gamma^2}{d_b^2 - \gamma^2} = 5.4.$$

Therefore:

$$\delta f_a = -28 \cdot 8.3 = -232 \text{ cps}$$
$$\delta f_b = +28 \cdot 5.4 = 151 \text{ cps}$$

Analogically, assigning ourselves other damping magnitudes, we obtain the corresponding value of δf. For large damping values, the magnitude of d will approach 1.

The filter's damping in the pass-through band is clear from the curves given in Fig.8. At the middle frequency:

$$b_{noise} = 3 \cdot \frac{n}{Q} = 3 \frac{1000}{50\,000} = 0.06 \text{ nep}$$

Frequencies $f_0 - 20$ cps. correspond to η equals 0.71, whereas $f \pm 10$ cps. correspond to η equals -0.356. Thus, the characteristics of the filter satisfy the stated requirements.

Further, we determine the unit capacitance C from formula 51:

$$C = 23.8 \text{ f}$$

Consequently:

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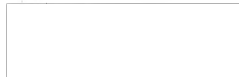
$$R = \frac{1}{\omega_0 C} = \frac{1}{2\pi \cdot 56\,000 \cdot 23.8} = 120\,000 \ Ohm$$

Therefore:

$$R_1 + R_3 = R_3 + R_4 = 120\,000 \ ohm$$

Let us assume that the anode-grid or grid-cathode capacitance in the inter-cascade connection equals ~15f. Then, adopting $R_1/R = R_4/R = 0.1$, we obtain a capacitance, recalculated into the filter, equal to 1.5 f. Therefore, we can take the additional capacitance ΔC_{13} as equal to ~12 f.

The damping by voltage between input 1 and output 4 of the inter-cascade connection, according to formula 52, equals:

$$B = 2.9 + b_p + b_{noise}$$

The additional damping introduced by the inter-cascade connection circuit is extinguished by amplification of the tube.

The recalculated capacitance comprises a small part of ΔC_{13}, which guarantees stability of the characteristics in the pass-through band when we change the tube. Fig.10.

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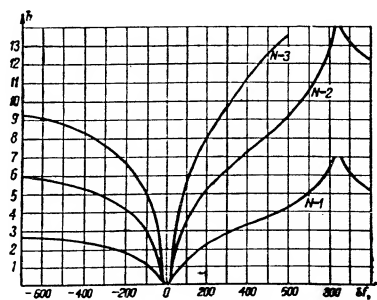
In Fig.10, we give the characteristics of the operating damping of the filter for N equals 1, 2, 3.

The measured characteristics coincide extremely well with the calculated ones. Article received by the Editors on March 20, 1953.

FOOTNOTES

1) If the middle frequency were not situated within the pre-assigned limits, we would evidently be able to produce the resonator with a somewhat altered frequency magnitude without noticeable changes in the remaining parameters.

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A.M.STRASHKEVICH and A. S. REIZLIN

ON THE ELECTRONO-OPTICAL ACTION OF GRID SYSTEMS

(A study is made of the electrono-optical action of a triode, the study being of particular interest for calculating and constructing cathode-ray tubes.

It is shown that it is essentially incorrect to proceed from formulas for the individual grid cell. A more precise version is given for the customary formula for the distribution of potential in a flat triode. A study is made of the relationship between the position of the focus and the electrical and geometrical parameters of the system.)

In recent years we have seen the great benefits of applying methods of electronic optics to the construction and calculation of various devices used in technical electronics. However, literature on the subject is still highly undeveloped with regard to the question of the electrono-optical action of widely used grid constructions, which is of special significance in cathode-ray tubes, vacuum tubes with secondary emission,etc.

Study of only the "distant" grid field permits us to calculate the device in terms of the current; to study the focusing action we can no longer restrict ourselves to examining "ideal" grids, but we must proceed from the potential distribution while also incorporating the "near" field of the real grid (terminology from Bibl.1).

As we know, the calculation of any multi-electrode tube is essentially the calculation of a series of equivalent triodes (1). This is valid not only in relation to the calculation of a device by the current, but also in relation to the calculation of the electrono-optical action of each grid individually. In recent times we observe a tendency toward producing flat-electrode tubes (2) which have a number of practical advantages owing to the identical density of the electron stream. It is therefore of interest, first of all, to examine the

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electrono-optical action of precisely such a flat triode. (The results can be directly transferred to apply to some cylindrical units).
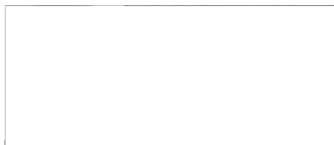
Fig.1
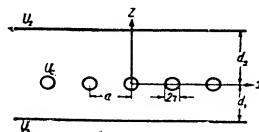
In Fig.1 we show a flat triode whose grid we will regard as consisting of an infinitely large number of parallel wires having round section. If the radius r of the section is small in comparison with a, d_1 and d_2, then the potential distribution can be presented in the following form:

$$\varphi(x, z) = A \ln \frac{\kappa_0}{\sin^2 \frac{\pi}{a} x + \sh^2 \frac{\pi}{a} z} + B(d_1 + z) + C, \tag{1}$$

where, when $d_1 \leqslant d_2$

$$A = -\frac{d_1(U_2 - U_1) + (d_1 + d_2)(U_1 - U_c)}{d_2 \ln \frac{\kappa_2}{\kappa_1} - (d_1 + d_2) \ln \frac{\kappa_2}{\kappa_0}},$$

$$B = \frac{U_2 \left(\ln \frac{\kappa_2}{\kappa_1} - \ln \frac{\kappa_2}{\kappa_0} \right) + U_1 \ln \frac{\kappa_2}{\kappa_0} - U_c \ln \frac{\kappa_2}{\kappa_1}}{d_2 \ln \frac{\kappa_2}{\kappa_1} - (d_1 + d_2) \ln \frac{\kappa_2}{\kappa_0}},$$

$$C = \frac{(d_1 + d_2) \left(U_c \ln \frac{\kappa_2}{\kappa_1} - U_1 \ln \frac{\kappa_2}{\kappa_0} \right) - d_1 U_2 \ln \frac{\kappa_2}{\kappa_1}}{d_2 \ln \frac{\kappa_2}{\kappa_1} - (d_1 + d_2) \ln \frac{\kappa_2}{\kappa_0}},$$

while when $d_1 \geqslant d_2$

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$$A = - \frac{d_1 (U_2 - U_1) + (d_1 + d_2)(U_1 - U_c)}{d_1 \ln \frac{\kappa_1}{\kappa_2} - (d_1 + d_2) \ln \frac{\kappa_1}{\kappa_0}},$$

$$B = \frac{(U_c - U_1) \ln \frac{\kappa_1}{\kappa_2} + (U_1 - U_2) \ln \frac{\kappa_1}{\kappa_0}}{d_1 \ln \frac{\kappa_1}{\kappa_2} - (d_1 + d_2) \ln \frac{\kappa_1}{\kappa_0}},$$

$$C = U_1;$$

where U_1, U_c and U_2 are the potentials of the cathode, grid and anode,

$$\kappa_0 = \sin^2 \frac{\pi}{a} r, \quad \kappa_1 = \mathrm{sh}^2 \frac{\pi}{a} d_1, \quad \kappa_2 = \mathrm{sh}^2 \frac{\pi}{a} d_2.$$

The potential distribution in the system depicted in Fig.1 is usually determined by means of an approached formula (2):

$$\bar{\tau}(x, z) = \frac{Q_1}{2} \ln \left(1 + e^{\frac{4\pi z}{a}} - 2e^{\frac{2\pi z}{a}} \cos \frac{2\pi x}{a} \right) + Q_2 \frac{2z}{a} (z + d_1), \qquad (2)$$

where:

$$Q_1 = \frac{U_2 + U_c \left(1 + \frac{d_2}{d_1} \right)}{\frac{2\pi d_2}{a} - \left(1 + \frac{d_2}{d_1} \right) \ln \left(2 \sin \frac{\pi r}{a} \right)},$$

$$Q_2 = \frac{U_c \frac{d_2}{d_1} - U_2 \frac{a}{2\pi d_1} \ln \left(2 \sin \frac{\pi r}{a} \right)}{\frac{2\pi d_2}{a} - \left(1 + \frac{d_2}{d_1} \right) \ln \left(2 \sin \frac{\pi r}{a} \right)},$$

(where U_1 equals 0).

Formulas 1 and 2 satisfy the Laplace equation (it is easily shown that the variable magnitudes in them coincide), but formula 1 is more accurate since in its development the values of the coefficients are obtained proceeding from the requirement of satisfying the boundary conditions on all electrodes with the degree of exactness necessary for practice.

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This is illustrated by the table, in which φ_0 signifies the pre-assigned potential value on the electrodes, and φ_1 and φ_2 the corresponding potential values calculated from formulas 1 and 2. As we see from the table, the relative error given by formula 1 on the electrodes is an order smaller than that given by formula 2. In the table, the percent of error is indicated for each value of φ_1 and φ_2; the cathode potential U_1 equals 0, the anode potential U_2 equals 1; d_1 equals d_2 equals d.

See next page for table.

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$\dfrac{d}{a}$		$x=r$ $s=0$	$x=0$ $s=-r$	$x=0$ $s=r$	$x=\dfrac{a}{2}=5r$ $s=d$	$x=0$ $s=d$
3,0	φ_0	$-0,0200$	$-0,0200$	$-0,0200$	1,0000	1,0000
3,0	φ_1	$-0,0200$ $(0,0\%)$	$-0,0183$ (9%)	$-0,0183$ (9%)	1,0000 $(0,0\%)$	1,0000 $(0,0\%)$
3,0	φ_2	$-0,0188$ (6%)	$-0,0326$ (68%)	$-0,0019$ (90%)	0,9222 $(7,8\%)$	0,9222 $(7,8\%)$
2,5	φ_0	$-0,0200$	$-0,0200$	$-0,0200$	1,0000	1,0000
2,5	φ_1	$-0,0200$ $(0,0\%)$	$-0,0180$ (10%)	$-0,0180$ (10%)	1,0000 $(0,0\%)$	1,0000 $(0,0\%)$
2,5	φ_2	$-0,0177$ $(11,5\%)$	$-0,0340$ (70%)	$+0,0024$ (112%)	0,9245 $(7,6\%)$	0,9245 $(7,6\%)$
1,0	φ_0	$-0,0200$	$-0,0200$	$-0,0200$	1,0000	1,0000
1,0	φ_1	$-0,0200$ $(0,0\%)$	$-0,0154$ (23%)	$-0,0154$ (23%)	1,0000 $(0,0\%)$	0,9884 $(1,2\%)$
1,0	φ_2	$-0,0147$ $(26,5\%)$	$-0,0522$ (161%)	$+0,0315$ (257%)	0,9308 $(6,9\%)$	0,9303 (7%)

Proceeding from correlation 1 we can study the electron-optical properties
of the system, assuming that the influence of the space charge near the cathode
is in practice insignificant (3) with regard to the trajectory of the electrons.

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In curve 1 of Fig.2 we give the relationship of the closing potential U_{c0}, calculated from formula 1, to d/a (where U_1 equals 0, d_1 equals d_2 equals d). We define U_{c0} as the smallest (in terms of the absolute magnitude) negative grid potential at which not a single electron (with an initial energy of zero) pass through the system, i.e. the potential at point z equals 0, x equals a/2 will be equal to zero. Curve 2, calculated from the formula adopted for computing the permeability of tube grids and giving good coincidence with experimental results (4,5), lies close to curve 1. Curve 3, calculated from the inexact formula 2 where d_1 equals d_2 equals d, produces a large deviation (up to 100%) from curve 2.

Fig.2.   Fig.3.
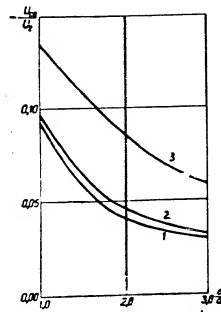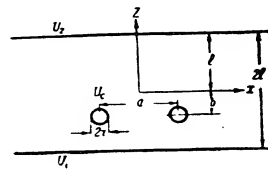

The collecting or dispersing action of the field is determined by the sign of $d^2\varphi(\frac{a}{2}, z)/dz^2$. From formula 1 we get:

$$\frac{d^2\varphi\left(\frac{a}{2}, z\right)}{dz^2} = -2A\left(\frac{\pi}{a}\right)^2 \frac{ch^2\frac{\pi z}{a}}{\left(1 + sh^2\frac{\pi z}{a}\right)^2}. \qquad (3)$$

From formula 3 it follows that the field of system Fig.1 over its entire

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extent, possesses only a collecting or only a dispersing action. (The same result can obviously be obtained from formula 2). In this resides the principal distinction (not noted in literature) between the field of such a system and the field of an individual grid cell (Fig.3), which usually has both a collecting and dispersing region.

For the potential distribution in the field of system Fig.3 with a small-diameter wire, we can obtain from the formulas of Bibl. 6:

$$\varphi(x, z) = \frac{U_c - U_1 - (U_2 - U_1)\frac{l-d}{2l}}{\ln T(c-r, -d)} \ln T(x, z) + U_1 \frac{(U_2 - U_1)(l+s)}{2l}, \quad (4)$$

where

$$T(x, z) = \frac{\left[\operatorname{ch}\frac{\pi(x-c)}{2l} + \cos\frac{\pi(z-d)}{2l}\right]\left[\operatorname{ch}\frac{\pi(x+c)}{2l} + \cos\frac{\pi(z-d)}{2l}\right]}{\left[\operatorname{ch}\frac{\pi(x-c)}{2l} - \cos\frac{\pi(s+d)}{2l}\right]\left[\operatorname{ch}\frac{\pi(x+c)}{2l} - \cos\frac{\pi(s+d)}{2l}\right]}.$$

On axis z we will have:

$$\varphi(0, z) = \frac{2\left[U_c - U_1 - (U_2 - U_1)\frac{l-d}{2l}\right]}{\ln T(c-r, -d)} \ln \frac{\operatorname{ch}\frac{\pi c}{2l} + \cos\frac{\pi(z-d)}{2l}}{\operatorname{ch}\frac{\pi c}{2l} - \cos\frac{\pi(z+d)}{2l}} + U_1 + \quad (5)$$

$$+ \frac{(U_2 - U_1)(l + z)}{2l}.$$

In Fig.4 we give graphs of $d^2\varphi(\frac{a}{2}, z)/dz^2$ (curve 1) and $d^2\varphi(0, z)/dz^2$ (curve 2); the former is calculated for system Fig.1 where d_1 equals d_2 equals 30r, a equals 10r, and the latter for system Fig.3 where b equals 0, 1 equals 30r, a equals 2c equals 10r. In both cases U_1 equals 0; U_c equals -0.02; U_2 equals 1.
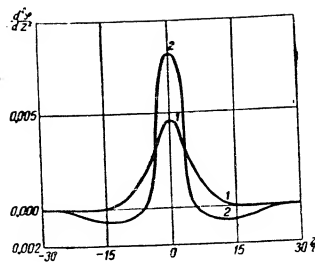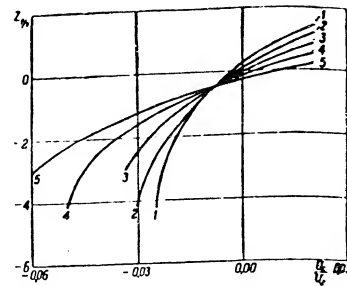
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Fig.4.   Fig.5.

The condition for the Fig.1 field collecting action's transition to a dispersing action upon increase of U_c is obtained from formula 3. Where:

$$\frac{d_1}{d_1 + d_2} < \frac{U_1 - U_c}{U_2 - U_1} \tag{6}$$

we have a collecting action, and with the reverse inequality sign we have a dispersing action.

For system Fig.1, applying the method described (for axial-symmetrical fields) in Bibl.7, and taking into account its necessary alteration for the field of a cylindrical lens (8), we obtain the focus coordinates z_f given in Figures 5, 6 and 7, for different geometrical and electrical parameters of the system. The position of the focus is of essential practical importance, for example, when calculating the optimum position of the screen grid in a beam-power tube. In Fig.5, curves 1, 2, 3, 4 and 5 give the relationship of z_f to U_c/U_2 for d_1 equals d_2 equals 30r, 25r, 20r, 15r and 10r respectively.

In Fig. 6 we give the relationship between z_f and d_2/a where U_c equals 0.00

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(curve 1) and where $U_c = -0.02\ U_2$ (curve 2) for $d_1 = a = 10r$. In Fig.7 we show the relationship between z_f and $\dfrac{d}{a}$ (where $d = d_1 = d_2$) for potentials of $U_c = -0.02\ U_2$ (curve 1), $U_c = 0.00$ (curve 2) and $U_c = +0.02 U_2$ (curve 3). Calculation of all relationships shown in Fig. 5, 6 and 7 was done on the assumption of $U_1 = 0$; $a = 10r$. As we can see from the graphs, the lower the grid's potential, the closer the effective focus moves toward the cathode, and this the faster, the denser the grid.

Comparison of the above results with calculations done by the same method for a separate grid cell (Fig.3) allows us to make a quantitative estimation of the error involved when substituting the action of the grid's field with the action of individual grid cells. Thus, assuming $a = 10r$, $U_1 = 0$, $U_c = -0.02U_2$ for the system Fig.1 where $d_1 = 10r$, $d_2 = 30r$ we get $z_f = -2.7r$; where $d_1 = d_2 = 30r$, $z_f = -2.2r$. But for an individual cell under the same corresponding geometrical parameters, i.e. where $b = 10r$, $l = 20r$, $a = 2_c = 10r$, we get $z_f = -1.3r$ (counting off zf from the grid), and where $b = 0$, $l = 30r$, we get $zf = 12.7r$.
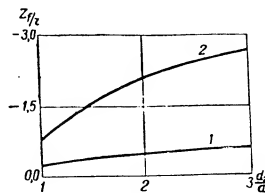
Fig.6
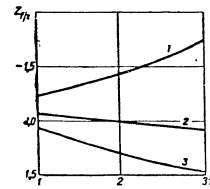
Fig.7

Whereas, when approximately characterizing the electrono-optical action of the grid, we can proceed from the field of an individual cell for $\dfrac{d_1}{a} < 1$, the size of the error becomes inadmissibly large when we deal with larger values of $\dfrac{d_1}{a}$.

Article received by the Editors on April 16, 1954.

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ON THE 60th BIRTHDAY OF A.L.MINTZ.

The Soviet people note the 60th birthday and 35 years of scientic activity of corresponding member of the Academy of Sciences USSR, Alexander Lvovich Mintz.

A.L.Mintz devoted his research to many problems in high frequency technology. His most fruitful activity was carried out in the field of radiobroadcasting devices. We are all familiar with the rapid development which this technolo-, gical field has undergone in the last three or four decades. The majority of basic problems arising in this development road were reflected in the works of A.L.Mintz, as a researcher, a scientific organizer, an inventor and a builder.

The outstanding significance of his activity in the cause of developing Soviet radio engineering results from the fact that A.L.Mintz not only recognizes and sets about solving the most urgent problems in radio transmission, but also that with his characteristically broad scientific outlook and tenacity he was and is able to organize work toward the most rapid realization of the newest ideas and projects in advanced radio equipment.

A.L.Mintz was born in Rostov-on-the-Don on January 9, 1895. His independent scientific activity began while he was still a student in the Physico-Mathematics faculty of the Moscow University. He directly participated in the Civil War as the radio division commander of the 1st cavalry.

After the Civil War had ended, one of the major problems of military radio communication was conversion from spark stations to tube stations. A.L.Mintz took an active part in resolving this problem, working in the radio laboratory of the Higher Military Communication School, the president of which he became in 1921. Here he designed the first radio station working on tubes, which the Red Army adopted together with the full-scale incorporation of audio frequency a.c.

In 1923, A.L.Mintz became the head of the Scientific Research Institute of

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Red Army Communications, which was organised to study matters of military communication. Many of his works published from 1923 to 1925 deal with the construction of army radio stations, their power supply, tonal telegraphy, etc.

About this time, radio telephony and radiobroadcasting became the most vital questions in the world of radio transmitting devices. Together with I. G. Klyatskin, A. L. Mints developed a method for calculating generators, based on the linearization of tube characteristics, and also methods for calculating plate and grid modulation (1926-1928). These works were of vital important for that time since there was a growing need for an engineering approach to the designing of transmitters.

During the years 1924 to 1926, A. L. Mints constructed a number of radio-telephonic and broadcasting transmitters having powers up to 20 Kw and transmitting regularly. He raised transmission quality to a high level for those days, thanks to his study and improvement of all the links of the transmitting circuit.

A. L. Mints payed great attention to radio-telephonic modulation circuits. His grid modulation design was put to use in all radiobroadcasting stations employing grid modulation constructed by us during the 1930's.

In the middle of the 1920's, when short-wave radio took on great importance, A. L. Mints devoted a great deal of time to this question. Besides constructing a series of short-wave message transmitters and a short-wave radiobroadcasting transmitter, he organised special expeditions for observing and studying short-wave propagation. To solve the problem of frequency stabilization he used an electron tube which acted on the parameters of an oscillatory circuit and in this way varied its frequency.

In connection with the need to build, relying solely on the young Soviet industry, a powerful radiobroadcasting station, the VTsSPS, operating at 75 to 100 Kw, A.L.Mints was invited to work in industry, where he organized in the beginning of 1928 the Bureau of Power Radio Construction. In 1929 the station

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was already in operation. The successful fulfilment of this task was an important
step in the cause of developing native radio power. It was not only that this
station was, for its time, the most powerful in Europe; world leadership in the
power of radiobroadcasting transmitters belonged to our country even in preceding
years, thanks to the work of M. A. Bonch-Bruyevich and A. L. Mints. But in
planning and building this station we note those organizational principles and
that style of work that later assured the rapid development of Soviet high-power
radio and also the possibility of the rapid construction of unical radio equipment.
The designing of such equipment usually encompasses, in addition to purely radio
skills, an extremely wide knowledge of engineering questions. The incorporation
of previously untested radio systems, often with the application of new vacuum
devices, requires detailed theoretical and experimental work. Although the
action of the new equipment might have a great effect on other parts of installa-
tions, A. L. Mintz with this characteristic boldness and technical intuition,
was able, in the very first stages of work, to pick out such flexible solutions
that he was able with a minimum of technical risk to carry out designing, plan-
ning, construction and parts ordering all at the same time, despite the fact that
he was faced with many and sometimes highly important unknowns.

The next great stride in the development of transmitting devices was the
designing and construction of the first radiobroadcasting long-wave station,
operating at 500 Kw (1931-1933). In this station A. L. Mintz introduced the
system of joint block work, later developed into the system of modulator-generator
blocks.

The problems arising in connection with building this station, just as the
more general problems raised by the development of transmitting devices in those
days, demanded profound studies and designs relating to the questions of generator
frequency stabilization, stabilization of the generator's working conditions, the

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electrical and power characteristics of transmitters, the construction of transmitters for different wave bands, antenna devices, power supply, special materials,etc. To solve this aggregate of problems, A.L.Mintz organized a multi-branch radio laboratory for transmitting equipment and later a high-power radio plant. Here, beginning with 1930, under his supervision were conceived, designed and constructed a large number of powerful radio stations, trunk-line transmitters, special radio stations and powerful radio centers. In this same institution there arose a large collective of practical workers and a large number of highly expert specialists.

While supervising the complex work of this organization, A. L. Mintz published a number of works relating to the construction of high-power transmitting installations, to increasing their efficiency, to dismountable tubes,etc.
The matters of calculating antennas, of selecting the best antenna parameters for powerful radio stations, of directive long-wave antennas and rigid short-wave band antennas (such antennas were used in 1936-1938 at the then most powerful 120-Kw short-wave radio station), were elucidated in a number of works by A. L. Mintz, beginning with 1922. We will not dwell here on his research dealing with radio measurements, rapid-action and letter-printing radio telegraphy and other similar fields of radio engineering.

During the 2nd World War, A. L. Mintz headed the construction of a super-powerful radio station. Put into operation in 1943, it is to this day the most powerful in the world.

After this he exerted his errots in new fields of applying high frequency, that were of importance in developing the very newest fields of modern physics.

A. L. Mintz combined his own research work with pedagogical activity with the training of young specialists and with social activity. Even at present, together with his basic activity in the Academy of Sciences USSR, he takes an active part in the work of the scientific soveits of a number of institutes, in the editorial

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board of "Radio Technology", in the presidium of the A.S.Popov Society,etc.

The activity of A. L. Mints has often been rewarded by the government A.L. Mitz is twice a Stalin prize laureate. In 1950 the presidium of the Academy of Sciences USSR awarded him the A.S.Popov gold medal.

We would like to wish Alexander Lvovich Mintz many more years of fruitful work devoted to the well-being of our homeland!

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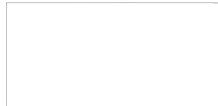
SCIENTIFIC CONFERENCE ON TELEVISION BROADCASTING

In December of 1954, there was held in Leningrad a scientific conference for exchange of experience in the exploitation of television centers and in the technology of television broadcasting, the conference being organized by the Leningrad and Ukrainian sections of the All-Union Scientific and Technical A.S.Popov Society of Radio Engineering and Electrical Communication.

The conference heard and discussed twenty-six reports and a number of informational announcements devoted to the exploitation of the country's television centers, color television, problems of television broadcasting, industrial applications of television, distant television reception and other questions in television technology.

In a report by the head engineer of the Moscow Television Center (MTTs), V.B. Renard, on the theme "Working Experience of the MTTs", the conference learned that the reconstruction of the MTTs carried out in 1948 made it possible for them to greatly increase the number of their television broadcasts. To do this they increased the number of studios to two, the number of studio channels to five, the number of movie channels to three and set up a mobile television station.

In the process of exploiting the MTTs station, they came up against a number of defects in the television apparatus of the studios: complex structure and cumbersome layout of the synchrogenerator; unsatisfactory installation of lighting equipment; unsuccessful installation of the intermediate amplifier; poor installation of the iconoscope; etc. All these defects must be carefully studied and eliminated in the future.

In the reports given by N. N. Sagarda and S. L. Kopansky, "Working Experience of the Kiev Television Center (KTTs) and "Working Experience of the U-H-F radio Station of the KTTs", the conference learned of many advantages of u-h-f broadcasting: high exploitation and quality indices of its antenna-feeder system and filter system, and also the successful system of line voltage stabilization.

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Among the shortcomings of the KTTs radio station are: narrow frequency band
(4.5 Mc); the unsuccessful constructional aspects of the mutual reservation
blocks and also their poor shielding; the poor installation of the control panels,
and also the layout of knob commutation.

Many of the reports to the conference were devoted to matters of color tele-
vision, but they were in general too abstract. The conference noted our slowness
in resolving the problems of color television and pointed out the necessity for
intensifying work in this direction. It is interesting to note that the discus-
sion of this question in connection with the report made by Prof. V. L. Kreitser
at the All-Union Scientific Session of the Society in 1954 also ran short of a
clear solution in the matter of defining a scientific and technical policy toward
the development of color television in the USSR. This shows once again that
these problems are still being paid too little attention.

In the report "Color Television System" made by Y. G. Minenko, the conference
heard descriptions of three systems: one with color alternation by fields, which
has been given the conditional name of consecutive color transmission system and
two systems of simultaneous color transmission.

Evaluating the qualities of the consecutive color system, the speaker came
to the conclusion that this system gave little promise. However, this conclusion
was insufficiently demonstrated and was met with strong objections on the part of
industry's representatives.

The speaker gave a detailed description of the system of simultaneous color
transmission with quadrature subcarrier modulation situated in the brightness
signal spectrum.

A method for producing quadrature modulation for transmitting several in-
dependent programs where the radio station has a pre-assigned restricted frequency
band was proposed by corresponding member of the Academy of Sciences USSR,

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A. A. Pistolkors in 1933. In 1939, Prof. E. G. Momot created an experimental radiobroadcasting transmitter with quadrature modulation capable of simultaneous transmission of two programs. The question is now being debated as to the applicability of quadrature modulation to color television with simultaneous transmission of the three basic colors. The report gave a fairly detailed account of the possibility of accomodating three independent television channels. The conference was given the necessary data and descriptions of circuits. When accomodating television channels in this way, it is possible to realize a color television system with simultaneous color transmission having sufficient clarity without it being necessary to greatly enlarge the radio station's frequency band.

In the report of V. G. Semyonov, the conference was informed of the basic features and technical characteristics of the color television receiver "Raduga".

I. A. Alekseyev, in his report "Modern Picture Tubes for Color Television", pointed out that three types of tubes are of greatest interest: a) the picture tube with a white-luminescence screen in conjunction with a rotating disc or drum having color filters; b) the color tube with a mosaic screen and a distributor (shade) lattice; c) the picture tube with a ruled screen and control grids. A detailed description of these tubes was given.

The picture tube with a white-luminescence screen in conjunction with a rotating disc with light filters should possess elevated screen brightness to compensate light losses in the color filters. The Soviet tube created for this purpose, the 18LK6B, possesses a luminescence brightness of 30 millistilb at a plate voltage of 15 Kv and an average ray current of 100 microamperes. The tube is distinguished by a slightly larger conical part than ordinary ones and by the application of screen metallization to raise its light output. The cathode-luminescent screen is made from a mixture of sulphide luminophors of red, green and blue luminescence.

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I. K. Malakhov, in a report entitled "Modern Television Transmitter Tubes", paid particular attention to the operating characteristics of modern tubes of the type LIZ, LI17, LI18, LI19 and LI20. He gave detailed descriptions of their sensitivity, resolving power, spectral characteristics, signal interference ratio and possibilities for use in color image transmission.

Non-linear distortions are particularly noticeable in iconoscopes with image transfer, in which we observe inconstancy of the contrast coefficient within the limits of the transmitted range of brightnesses. In tubes with two-sided targets it is possible to control the light characteristic through careful selection of the target's working conditions. The complete possibilities have not yet been studied for tubes with photoconducting targets in which the form of the light characteristic can be varied through the choice of the target's material.

The fundamental defects in iconoscopes with image transfer are, in the opinion of the speaker, bound up with the presence of a "black spot". In tubes with a two-sided target, the fundamental defect is contamination of the working surfaces and irregularity of the signal along the field. Tubes with photo-conducting targets are free from the latter defect.

In the discussion, the backwardness of the vacuum tube industry was pointed out, especially with regard to the small number of tubes put out, their short-livedness, the unstable standardization of their characteristics and parameters, their high cost and the low clarity of images in picture tubes, the limited nature of receiver-amplifier tubes,etc. All this shows that too little attention is being paid to these matters in our industry.

In the report of G. Z. Besidsky, "The Small Television U-H-F Radio Station", the conference heard about a station that might be used in many cities for relaying television programs and for establishing small television centers in the regional cities of our country. Until the present, however, these stations have not

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been installed and mainly through the fault of the radio industry, which has not put into practice their mass production. This obliges many organizations to construct such stations using amateur methods.

The initiative of Kharkov radio amateurs, who built the first Soviet amateur television center, inspired amateurs in other cities as well. Thus, amateur television centers were built in Gorky, Tomsk, etc. In a report given by D. Kryzhanovsky, "The Small Television Center of Gorky", an account was given of the structure, technical data and electrical circuity of the separate blocks. The operational results and enlargement perspectives of the center were also pointed out.

The report of A. G. Kondratyev on "The Industrial Applications of Television" was found unsatisfactory by the conference despite the importance of the theme. The report was extremely brief and only the regions of possible use of television in industry and agriculture were cited and not the specific results gained by the speaker in his work under prof. P. V. Shmakov.

We know that television is becoming more and more useful in underwater research, in geological investigations, in planning oil wells, in transmitting X-ray images, in metallography, etc. At present, the further development of television in industry is being held up mainly by the absence of specialized organizations that might create the necessary equipment.

I. F. Nikolayevsky, in his report "Ways and Means of Suppressing Television Interference", informed the conference that after an investigation of television receivers operating in various parts of Moscow during 1952 and 1953, the following distribution of interferences was found to exist: from radio stations —62%; from medical apparatus — 12%; from automotive and trolley traffic — 12%; from low-power electric motors — 7%; from electric welding machinery — 2%; from the signal reflection by passing airplanes — 0.5%; and from other interference sources (electric bells, electric signs, etc) — 0.5%. A survey was given of

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the frequency characteristics of interferences, the ways in which they penetrate a television set, the types of image and sound distortion brought about by interferences, the ways and means of eliminating interference, the experience in applying anti-interference devices and future possibilities in this field. Sample anti-interference devices were demonstrated.

In the comments relating to I. F. Nikolayevsky's report it was pointed out that anti-interference attachments for television sets were being incorporated in too limited a fashion.

The conference also devoted attention to questions of longrange television reception. It was pointed out that the question of long-range and extra-long-range reception is being given very little attention at present even though a knowledge of long-range and extra-long-range reception might give us additional information with regard to the theory of radio wave propagation.

It is evident that the proposed long-range reception might not assure the necessary regularity for the daily use of television sets. However, in a number of instances, successful reception might be attained. This fact is confirmed by the reception of KTTs broadcasts in Gomel (225 kilometers), Romny (210 km), Kirovograd (248 km), Vinnitsa (195 km), Rovno (310 km), Poltava (300 km), Vapnyarka (247 km) and at a number of other points (*).

At all the above points, reception is basically made with rhombic antennas situated several stories high and television sets of the type "Leningrad T-2" with high-frequency 2-to-4-stage amplifiers.

The sound part of the KTTs station is clearly heard in Kharkov, Sumy, Minsk (390 km) and other cities. It can be assumed that with the proper measures it would be possible for the KTTs picture to be received at some points in these cities.

Also of interest are cases of extra-long-distance reception. For instance, in

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Poltava they received broadcasts from an Italian television center; in Nezhina there is periodic reception from a number of foreign stations. These instances of extra-long-distance reception are reported by chance. If we were to organize regular studies on television reception, we might establish that certain regular phenomena exist.

The conference displayed great interest in the talk given by engineer Beno (Czechoslovakia). He reported on the results of candidate of technical sciences, Mrazek, who recorded repeated instances of reception from the MTTs, LTTs and other television centers of the world at 40 km from Prague. As a rule, the field intensity did not depend on the orientation of the receiver dipole, and in a number of cases broadcasts from the MTTs covered up the signal intensity created by the Prague television station. In their turn, radio signals from the Prague transmitter creates complications in the field intensity from the MTTs in Voronezh, as the conference was subsequently informed by P. Trifonov.

The conference's resolutions point out the necessity for designing a more effective color television system, improving the exploitation of television centers, increasing production and improving the quality of vacuum tubes, etc. It was particularly emphasized that we need a magazine to deal with questions of television, electronics and related matters. The need was also expressed for holding annual conferences on matters of television technology, as well as specialized conferences on individual television problems.

FOOTNOTE

*In parentheses are the distances between the television center and the reception point.

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NEW BOOKS

I. I. Grodnyev and V. V. Sokolov, "COAXIAL CABLES", Svyaz'izdat, M., 1954., 226 pages, 7 rubles 40 kopeks.

The book examines the theory of coaxial cables, the influence of constructional irregularities on the quality of electric signal transmission, the influences between separate coaxial pairs and their shielding, electrical measurements (particularly through impulse methods), the structure of coaxial cables, and also practical question of construction, installation and exploitation of cable lines containing coaxial pairs. The book is intended for engineering workers involved in the planning, construction and exploitation of cable lines, and for students in advanced courses at electric communication institutes.

B. V. Bulakov, OSCILLATIONS, Gostekhteoretizdat, M., 1954, 891 pages, 29 rubles 40 kopeks.

The first part of the book contains the fundamentals of matrix and operational calculus and an exposition of some questions in analytical mechanics that are closely bound up with the theory of oscillations.

The second part of the book examines free and forced oscillations of systems with one degree of freedom, mainly non-linear systems.

The third part examines: internal and forced oscillations of systems with many degrees of freedom; passive systems; linear regulated systems; the theory of linear systems with periodic coefficients; the theory of oscillations of non-linear systems with many degrees of freedom.

P. V. Shmakov, FUNDAMENTALS OF COLOR AND THREE-DIMENSIONAL TELEVISION, Izd. "Sovetskoye radio", M., 1954, 303 pages, 10 rubles 70 kopeks.

The first part of the book presents an exposition of the physical foundations

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of color television and of color calculations in color television systems; the
author gives a classification of such systems and the basic principles of their
construction, as well as a description of tubes for color television.

In the second part, devoted to three-dimensional (stereoscopic) television, the
author discusses the necessary conditions for the three-dimensional reproduction of
images, and also systems for realizing three-dimensional reproduction. The con-
cluding chapter is devoted to the question of three-dimensional color television.

A. A. Rizkin, FUNDAMENTALS OF THE THEORY OF AMPLIFYING CIRCUITS, Second Edition,
Iad. "Sovetskoye radio", M., 1954, 439 pages, 13 rubles 50 kopeks.

The first five chapters give the general methods for analyzing amplifying
circuits, and also a survey of selective amplifiers, audio frequency amplifiers
and amplifiers with feedback. The sixth chapter is devoted to wide-band ampli-
fiers, complex circuits for their correction, and also the internal noises of
amplifiers. Chapters 7 and 8 discuss the operating principles of impulse amplifi-
ers and some circuits. In chapters 9, 10 and 11 the author gives a description of
d.c. amplifiers, power audio amplifiers working under various conditions, and
also tubeless amplifiers -- crystal and magnetic.

S. G. Ginsburg, METHOD OF SOLVING PROBLEMS OF TRANSIENT PROCESSES IN ELECTRICAL
CIRCUITS, izd. "Sovetskoye radio", M., 1954, 252 pages, 7 rubles 90 kopeks.

The book examines various methods for solving problems dealing with transient
processes in linear electrical circuits. The author demonstrates the applica-
tion of the classical method, the operational method (particularly, the Heaviside
formulas when connecting into constant and harmonic voltage under zero starting
conditions), superposition integrals and superposition methods. The last chapter
is devoted to the spectral method and its application to the study of transient
processes.

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